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Regular Article

Hyperfine spin-spin interaction and Zeeman effect in the pure bound field theory

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Abstract. We carry out a joint analysis of the hyperfine spin-spin splitting (HFS) and the Zeeman effect in the framework of Pure Bound Field Theory (PBFT) we recently suggested (A.L. Kholmetskii *et al.* Eur. Phys. J. Plus **126** (2011) 33; **126** (2011) 35), where the PBFT corrections to the known results have a similar form due to the common physical origin of both effects. We consequently consider the hydrogen atom, positronium, muonium and muonic hydrogen atom and show that for the Zeeman effect in muonic hydrogen, the PBFT correction occurs measurable and its presence/absence can be subjected to an experimental test, which thus will be crucial for the verification of PBFT *versus* the common theory. Concurrently we derive the PBFT correction to the muon mass, which is cancelled in the joint analysis of HFS and Zeeman effect, but can be revealed in muon-spin-precession–resonance experiments with enhanced precision. As a result, we achieve better agreement between the estimations of the muon mass in different experiments. In addition, we have shown that the PBFT correction to the proton Zemach radius is one order of magnitude smaller than the measurement uncertainty and can be well ignored, unlike the case of the proton charge radius.

1 Introduction

Recently we suggested the Pure Bound Field Theory (PBFT) [1,2], which explicitly takes into account the non-radiative nature of the electromagnetic (EM) field of bound charges in the stationary energy states. We have shown that the absence of EM radiation for true quantum systems demands modifying dynamical parameters of electrically bound charges, in order to fulfill the momentum conservation law [1–3]. In particular, for the one-body problem, the rest mass of the electron m is replaced by $b_n m$, while the electric interaction energy U is replaced by $\gamma_n U$. Here b_n, γ_n are the specific coefficient of PBFT, which to the order α^2 are defined by the relationships [1]

$$b_n = 1 - (Z\alpha)^2/n^2, \qquad \gamma_n = \left(1 - (Z\alpha)^2/n^2\right)^{-1/2},$$

where α is the fine structure constant, Z is the atomic number, and n is the principal quantum number. This approach can be naturally extended to the quantum two-body problem, which prompted us to repostulate the Breit equations without the external field in the appropriate way, giving rise to the development of PBFT in the form of an effective theory [1,2]. Thus it does not touch the diagram technique of QED, though the modifications of dynamical parameters of electrically bound charges are also implied in QED of bound states [2]. Such a theory gives the same gross as well as fine structure of atomic energy levels, as those furnished by the conventional approach, for hydrogenlike atoms. However, the approach of PBFT does induce corrections to the energy levels at the scale of hyperfine interactions, which at once remove a number of subtle discrepancies between theory and experiment in the atomic physics. In particular,

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the PBFT corrections practically eliminate the available up-to-date deviation between theory and experiment for the $1S$–$2S$ interval and $1S$ spin-spin splitting in positronium [2].

In addition, PBFT gives the quantitative agreement between theory and experiment for the lifetime of bound muon in meso-atoms [4]. Another result of significant importance is the PBFT re-estimation of the proton charge radius derived via the $2S$-$2P$ Lamb shift for hydrogen and muonic hydrogen, which yields $r_p = 0.841(6)$ fm [2]. This value is about 4% smaller than the modern CODATA value $r_p = 0.876(6)$ fm [5], but perfectly agrees with the latest estimation of proton size via the measurement of $2S$-$2P$ Lamb shift in muonic hydrogen, *i.e.* $r_p = 0.84184(67)$ fm [6]. The intriguing point is that the latest experiments on electron scattering by protons, carried out after the publication of the work [6], give the proton charge radius still near the CODATA value [7,8]. At this point, one should stress that the PBFT corrections are applicable only to the non-radiating bound system, and this theory is not extended to freely moving particles. Hence, no PBFT corrections may emerge, when the scattering experiments are analyzed. Thus, the obvious contradiction between the results of spectroscopic measurements in muonic hydrogen [6] and scattering experiments [7,8] allow us to assume the presence of some non-accounted physical effect either in the processing of spectroscopic data, or in the processing of electron scattering data. In this connection further independent experimental verification of PBFT, which definitely supports spectroscopic data, seems topical for localization of search area for such non-accounted effect.

To be successful, any new theory of bound states like PBFT should provide not only improvement in the coincidence of the theory and experiment in cases where QED predictions show some discrepancy with experimental data, but also should not destroy the excellent agreement of the majority of QED calculations with measurements. As commonly known the formalism of quantum field theory is extremely constraining, after one selects a set of fields and a Hamiltonian/Lagrangian there are essentially no ambiguities left. This peculiarity does not allow to "tune" the Lagrangian in QED to achieve better agreement with experimental data for some parameters without destroying such an agreement for others. Therefore at the current stage of research it is very important to check PBFT results for all measured values in the precision physics of hydrogenlike atoms, since requirement for the absence (or negligible values) of PBFT corrections, in the cases where QED gives good agreement with observable data, should be considered as of equal importance with the cases where PBFT eliminates discrepancy between theory and experiment.

Within this framework in the present contribution, we continue to analyze hyperfine spin-spin splitting (HFS) in various hydrogenlike atoms, where the specific corrections of PBFT result not only due to the proper modification of dynamical parameters of bound particles, but also due to the re-estimation of their magnetic moments implemented via the Zeeman effect. Since the spin-spin interaction and Zeeman effect have a common physical origin, the PBFT corrections to the Hamiltonian of spin-spin interaction and to the Hamiltonian of interaction of spin with an external magnetic field acquires a similar character. Thus, in the case of Zeeman effect such corrections should be involved to the re-estimation of magnetic moments for constituents of atom. These circumstances determine the goal of the present paper, which is two-fold: to achieve better agreement between the estimations of the muon mass in different experiments and to suggest new independent tests of PBFT. One of them is the precise measurements of Zeeman effect in the muonic hydrogen atom; the other one is the repetition of muon-spin-precession–resonance experiments aimed to measure the muon magnetic moment and its mass with an enhanced precision.

The paper is structured as follows. For the convenience of the reader, in sect. 2 we reproduce the Breit equation without external field repostulated in PBFT framework [1], and present the method of its solution based on perturbation theory. In sect. 3 we separately consider the contribution due to spin-spin interaction and derive specific PBFT corrections to spin-spin interval for the nS state of hydrogenlike atom, which includes two components. One of them directly stems from the PBFT modification of dynamical parameters of bound particles for the quantum two-body problem, while the second component of the correction is related to the re-estimation of magnetic moments for both bound particles, based on the analysis of Zeeman effect in PBFT framework. Combining both kinds of correction, we achieve the PBFT expression for the HFS and analyze its implications in sects. 3.1–3.4 for various hydrogenic atoms. In particular, we show that for muonium these corrections cancel away, so that the calculated nS spin-spin interval has the same value in PBFT and in the common approach. In contrast, for positronium, both corrections are strongly unequal to each other, and the resultant PBFT correction is not vanished. With this correction we obtain better agreement of PBFT calculations with the measured $1S$ spin-spin interval, than in the common theory, as already pointed out in ref. [2].

The analysis of PBFT corrections we carry out allows revealing the most interesting case, where PBFT can be subjected to a new independent test: it is the Zeeman effect in a strong magnetic field for muonic hydrogen, as compared with the Zeeman effect in muonium (sect. 3.4). The recoil effects in these atoms differ from each other more than one order of magnitude, which causes the measurable difference of PBFT corrections to the Zeeman effects (about 10 ppm) that can be subjected to an experimental test. Concurrently we stress that PBFT corrections to the Zeeman effect imply the introduction of a related correction to the magnetic moment of muon and, correspondingly, to its rest mass. The predicted correction to muon mass exceeds the present measurement uncertainty, and we discuss possible ways for the new independent measurement of muon magnetic moment and its mass in sect. 3.5. Finally, we conclude in sect. 4.

2 Breit equation without external field in PBFT and its solution

For two bound charged particles with masses m, M and charges e, Ze, correspondingly, the Breit equation in the PBFT framework acquires the form [1]:

$$(H_{b0} + H_{b1} + \ldots + H_{b5})\, \xi\left(\boldsymbol{r}_m, \boldsymbol{r}_M\right) = W \xi\left(\boldsymbol{r}_m, \boldsymbol{r}_M\right), \tag{1}$$

where $\xi(\boldsymbol{r}_m, \boldsymbol{r}_M)$ is the wave function having 16 spinor components, $\boldsymbol{r}_m$, $\boldsymbol{r}_M$ are the position vectors for each particle, W is the energy, and the Hamiltonian components H_{bi} are determined by the equations based on the correspondent Breit's expressions [9]

$$H_{b0} = -\gamma_{mn}\gamma_{Mn}\frac{Ze^2}{r} + \frac{1}{2}\left(\frac{1}{b_{mn}m} + \frac{1}{b_{Mn}M}\right) p_b^2, \tag{1a}$$

$$H_{b1} = -\frac{1}{8c^2}\left(\frac{1}{{b_{mn}}^3 m^3} + \frac{1}{{b_{Mn}}^3 M^3}\right) {p_b}^4, \tag{1b}$$

$$H_{b2} = -\frac{Ze^2}{2b_{mn}b_{Mn}mMc^2}\frac{1}{r}\left({p_b}^2 + \frac{1}{r^2}\boldsymbol{r}\cdot\left(\boldsymbol{r}\cdot\boldsymbol{p}_b\right)\boldsymbol{p}_b\right), \tag{1c}$$

$$H_{b3} = -\gamma_{mn}\gamma_{Mn}\frac{\boldsymbol{r}\times\boldsymbol{p}_b}{r^3}\left(\frac{Ze^2\hbar}{2{b_{mn}}^2 m^2 c^2}\boldsymbol{s}_m + \frac{Ze^2\hbar}{2{b_{Mn}}^2 M^2 c^2}\boldsymbol{s}_M + \frac{Ze^2\hbar}{b_{mn}b_{Mn}mMc^2}\boldsymbol{s}_m + \frac{Ze^2\hbar}{b_{mn}b_{Mn}mMc^2}\boldsymbol{s}_M\right), \tag{1d}$$

$$H_{b4} = -\frac{iZe^2\hbar}{2c^2}\left(\frac{1}{{b_{mn}}^2 m^2} + \frac{1}{{b_{Mn}}^2 M^2}\right)\boldsymbol{p}_b\cdot\nabla\frac{1}{r}, \tag{1e}$$

$$H_{b5} = \frac{Ze^2\hbar^2\gamma_{mn}\gamma_{Mn}}{b_{mn}b_{Mn}mM}\left(-\frac{8\pi}{3}\left(\boldsymbol{s}_m\cdot\boldsymbol{s}_M\right)\delta(\boldsymbol{r}) + \frac{1}{r^3}\left(\boldsymbol{s}_m\cdot\boldsymbol{s}_M - 3s_{mr}s_{Mr}\right)\right), \tag{1f}$$

and the coefficients b_{mn}, b_{Mn}, γ_{mn}, γ_{Mn} are determined to the order $(Z\alpha)^2$ by the equations

$$b_{mn} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{M}{M+m}\right), \tag{2a}$$

$$b_{Mn} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{m}{M+m}\right), \tag{2b}$$

$$\gamma_{mn} = \left[1 - \frac{(Z\alpha)^2}{n^2}\frac{M^2}{(m+M)^2}\right]^{-1/2}, \tag{2c}$$

$$\gamma_{Mn} = \left[1 - \frac{(Z\alpha)^2}{n^2}\frac{m^2}{(m+M)^2}\right]^{-1/2}. \tag{2d}$$

Here $\boldsymbol{p}_b = \boldsymbol{p}_{mb} = -\mathbf{p}_{Mb}$ is the canonical momentum, $s_{mr} = \frac{\boldsymbol{s}_m\cdot\boldsymbol{r}}{r}$ ($\boldsymbol{s}$ is the spin operator), and $\boldsymbol{r} = \boldsymbol{r}_m - \boldsymbol{r}_M$.

The first operator H_{b0} of eq. (1) represents the PBFT counterpart to the conventional Schrödinger operator; H_{b1} is the relativistic expansion of H_{b0}; the operator H_{b2} takes into account the retardation in the interaction of two particles; H_{b3} describes spin-orbit interaction; H_{b4} is responsible for the contact interaction, and H_{b5} stands for spin-spin interaction.

We emphasize that the operators (1a)–(1f) stem from the corresponding common Breit operators (see, *e.g.*, [9]), modified by the PBFT replacements [1]

$$m \rightarrow b_{mn}m, \tag{3a}$$

$$M \rightarrow b_{Mn}M, \tag{3b}$$

$$U \rightarrow \gamma_{mn}\gamma_{Mn}U, \tag{3c}$$

$$\boldsymbol{E} \rightarrow \gamma_{mn}\gamma_{Mn}\boldsymbol{E}, \tag{3d}$$

$$\boldsymbol{B} \rightarrow \gamma_{mn}\gamma_{Mn}\boldsymbol{B}. \tag{3e}$$

By analogy with ref. [10], it is convenient to reduce the Breit equation to the Schrödinger-like type

$$\left[\frac{p_b^2}{2mb_{mn}} + \frac{p_b^2}{2Mb_{mn}} - \gamma_{mn}\gamma_{Mn}\frac{Ze^2}{r} - \frac{p_b^4}{8m^3{b_{mn}}^3c^2} - \frac{p_b^2}{8M^3{b_{Mn}}^3c^2} + U_b\left(\boldsymbol{p}_b, \boldsymbol{r}\right)\right]\psi(\boldsymbol{r}) = W\psi(\boldsymbol{r}), \tag{4}$$

where W is the energy, and the term $U_b(\boldsymbol{p}_b,\boldsymbol{r})$ is equal to

$$
\begin{aligned}
U_b(\boldsymbol{p}_b,\boldsymbol{r}) = &-\frac{\pi Z e^2\hbar^2}{2c^2}\left(\frac{1}{{b_{mn}}^2 m^2}+\frac{1}{{b_{Mn}}^2 M^2}\right)\delta(\boldsymbol{r}) + \frac{Ze^2}{2b_{mn}b_{Mn}mMr}\left(p_b^2+\frac{\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p}_b)\,\boldsymbol{p}_b}{r^2}\right)\\
&-\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{4{b_{mn}}^2m^2c^2r^3}(\boldsymbol{r}\times\boldsymbol{p}_b)\cdot\boldsymbol{\sigma}_m - \frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{4{b_{Mn}}^2M^2c^2r^3}(\boldsymbol{r}\times\boldsymbol{p}_b)\cdot\boldsymbol{\sigma}_M\\
&-\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{2b_{mn}b_{Mn}mMc^2r^3}\left((\boldsymbol{r}\times\boldsymbol{p}_b)\cdot\boldsymbol{\sigma}_M+(\boldsymbol{r}\times\boldsymbol{p}_b)\cdot\boldsymbol{\sigma}_m\right)\\
&+\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{4b_{mn}b_{Mn}mMc^2}\left(\frac{\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_M}{r^3}-3\frac{(\boldsymbol{\sigma}_m\cdot\boldsymbol{r})\,(\boldsymbol{\sigma}_M\cdot\boldsymbol{r})}{r^3}-\frac{8\pi}{3}\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_M\delta(\boldsymbol{r})\right).
\end{aligned}
\tag{5}
$$

Here $\boldsymbol{\sigma}$ stands for the Pauli matrix.

The obtained eqs. (4) and (5) differ from their common counterparts by the appropriate introduction of the correction coefficients of PBFT (b_{mn}, b_{Mn}, γ_{mn}, γ_{Mn}), which differ from unity in the order $(Z\alpha)^2$. Thus, in order to solve eq. (4), one can apply the perturbation theory approach, which is realized in the most convenient way via the substitution

$$
\boldsymbol{r}=\boldsymbol{r}'/\left(b_{mn}b_{Mn}\gamma_{mn}\gamma_{Mn}\right). \tag{6}
$$

This substitution allows us to present the Hamiltonian in eq. (4) as the sum of the Schrödinger-like term and perturbation. Indeed, taking into account that $p_b^2=-\hbar^2\nabla_r^2=-{b_{mn}}^2{b_{Mn}}^2{\gamma_{mn}}^2{\gamma_{Mn}}^2\hbar^2\nabla_{r'}^2$, we transform eq. (4) as follows:

$$
\left[-\frac{\hbar^2\nabla_{r'}^2 b_{Mn}}{2m}-\frac{\hbar^2\nabla_{r'}^2 b_{mn}}{2M}-\frac{Ze^2}{r'}+\frac{1}{b_{mn}b_{Mn}{\gamma_{mn}}^2{\gamma_{Mn}}^2}\left(-\frac{p_b^4}{8m^3{b_{mn}}^3c^2}-\frac{p_b^2}{8M^3{b_{Mn}}^3c^2}+U_b\left(\boldsymbol{p}_b,\boldsymbol{r}'\right)\right)\right]\psi(\boldsymbol{r}')=W'\psi(\boldsymbol{r}'),
\tag{7}
$$

where

$$
W'=W/\left(b_{mn}b_{Mn}{\gamma_{mn}}^2{\gamma_{Mn}}^2\right). \tag{8}
$$

The obtained eq. (7) complemented by the expressions (2), (5), (6) and (8) represents the basic equation for the quantum two-body problem within the framework of PBFT [1,2] and, as shown in ref. [1], eq. (7) yields the same gross and fine structure of the atomic energy levels, as the one furnished by the common approach.

Thus the corrections of BPFT to the common solutions of equations of atomic physics (still without spin-spin interaction and radiative corrections) may emerge at least in the order $(Z\alpha)^6$, which corresponds to the scale of hyperfine interactions. Here one should recall that eq. (7) itself, like the original Breit equation, is semi-relativistic, and it is valid to the order $(Z\alpha)^4$, where α is the fine structure constant. At the same time, the factors b_{mn}, b_{Mn}, γ_{mn} and γ_{Mn}, being explicitly determined to the orders $(Z\alpha)^2$ and $(Z\alpha)^4$ [2], allow us to analyze the specific PBFT corrections to the order $(Z\alpha)^6$, and the determination of these corrections is one of the important issues of PBFT.

We further notice that without the term of spin-spin interaction, eq. (7) determines the PBFT correction to the fine structure, which has the order of magnitude $(Z\alpha)^6 m/M$ and its terms scale as n^{-5} or n^{-6} [2]. Hence, in practice it occurs significant for the $1S$ state only. For the case of hydrogen, such a correction has the value 10.8 kHz for the $1S$ state [2], and along with radiative corrections to the ground state the Lamb shift plays an important role in the re-estimation of the proton charge radius extracted via the $1S$ Lamb shift, providing the value $r_p = 0.846(12)$ fm [2]. This value practically coincides with the proton size extracted via the classic Lamb shift already mentioned earlier ($r_p = 0.841(6)$ fm). The fine structure correction occurs also significant for the $1S$ state of positronium, where $m = M = m_e$ (m_e being the electron rest mass). Along with the PBFT correction to the annihilation term, it practically eliminates the available up to date discrepancy between theory and experiment with respect to $1S$–$2S$ interval [2].

In the next section we analyze separately the correction to the HFS, which, as we will see below, in PBFT framework should be complemented by the correction to the Zeeman effect, too.

3 Hyperfine spin-spin interaction and Zeeman effect in PBFT: Joint analysis

Now we evaluate the contribution of the hyperfine spin-spin interaction to the atomic energy levels in the PBFT framework, described by the operator

$$
\left(V_{\mathrm{PBFT}}(\boldsymbol{r})\right)_{s-s}=\frac{Ze^2h^2\gamma_{mn}\gamma_{Mn}}{4mb_{mn}Mb_{Mn}c^2}\left(\frac{\sigma_m\cdot\sigma_M}{r^3}-3\frac{(\sigma_m\cdot\boldsymbol{r})\,(\sigma_M\cdot\boldsymbol{r})}{r^5}-\frac{8\pi}{3}\left(\sigma_m\cdot\sigma_M\right)\delta(\boldsymbol{r})\right) \tag{9}
$$

(the last term of eq. (5)). Being expressed via r'-coordinates, this operator reads

$$(V_{\text{PBFT}}(\boldsymbol{r}'))_{s-s} = b_{mn}b_{Mn}{\gamma_{mn}}^2{\gamma_{Mn}}^2\frac{e^2h^2}{4mMc^2}\left(\frac{\sigma_m\cdot\sigma_M}{r'^3} - 3\frac{(\sigma_m\cdot\boldsymbol{r}')\,(\sigma_M\cdot\boldsymbol{r}')}{r'^5} - \frac{8\pi}{3}\,(\sigma_m\cdot\sigma_M)\,\delta(\boldsymbol{r}')\right), \tag{10}$$

where we have used eq. (6), the equality $\delta(\boldsymbol{r}) = {b_{mn}}^3{b_{Mn}}^3{\gamma_{mn}}^3{\gamma_{Mn}}^3\delta(\boldsymbol{r}')$, and also taken into account that, after the substitution of this operator in eq. (7), it acquires the factor $1/b_{mn}b_{Mn}{\gamma_{mn}}^2{\gamma_{Mn}}^2$. Designating

$$(V(\boldsymbol{r}'))_{s-s} = \frac{e^2h^2}{4mMc^2}\left(\frac{\sigma_m\cdot\sigma_M}{r'^3} - 3\frac{(\sigma_m\cdot\boldsymbol{r}')\,(\sigma_M\cdot\boldsymbol{r}')}{r'^5} - \frac{8\pi}{3}\,(\sigma_m\cdot\sigma_M)\,\delta(\boldsymbol{r}')\right) \tag{11}$$

(the common Hamiltonian of spin-spin interaction expressed via r'-coordinates), and substituting expressions (2a)–(2d) for the PBFT factors, determined with the sufficient accuracy $(Z\alpha)^2$, we obtain

$$(V_{\text{PBFT}}(\boldsymbol{r}'))_{s-s} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2mM}{(M+m)^2}\right)(V(\boldsymbol{r}'))_{s-s}\,. \tag{12a}$$

This relationship should be averaged with the non-relativistic wave function, which, as we mentioned above, has the known Schrödinger form in the $\boldsymbol{r}'$-coordinates. Hence eq. (12a) is also valid for the energy of spin-spin interaction

$$(W_{\text{PBFT}})_{s-s} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2mM}{(M+m)^2}\right)W_{s-s}. \tag{12b}$$

Here W_{s-s} stands for the spin-spin interval calculated in the common approach, which, like the source operator (12a), contains the ratios $g_m\boldsymbol{\sigma}_m/m$, $g_M\boldsymbol{\sigma}_M/M$ for both particles, which are determined experimentally by means of the Zeeman effect.

Here it is worth to stress that a direct measurable value in the Zeeman effect is the magnetic moment $\boldsymbol{\mu} = ge\boldsymbol{s}/m$ for each particle. However, in the derivation of specific PBFT corrections to Zeeman splitting and HFS, the quantities expressed via spin operators occur more convenient than the quantities expressed via magnetic moments. At the same time, in the comparison of experimental results with calculated data, the corresponding equations must be finally expressed through the magnetic moments of both particles as the directly measured values. This remark is important for the analysis carried out below.

As known, the operator of interaction of two bound particles (electron and nucleus) in the nS-state with a weak external magnetic field reads [9]

$$V_{mag} = g_m\frac{e\hbar}{2m}\,(\boldsymbol{s}_m\cdot\boldsymbol{B}) - g_M\frac{Ze\hbar}{2M}\,(\boldsymbol{s}_M\cdot\boldsymbol{B})\,, \tag{13}$$

where g_m, g_M are the g-factors for bound particles with the masses m and M, correspondingly. Being added to the Breit operator of eq. (3), along with the PBFT corrections (3a)–(3e), this operator acquires the form

$$(V_{\text{PBFT}})_{mag} = \frac{1}{b_{mn}b_{Mn}{\gamma_{mn}}^2{\gamma_{Mn}}^2}\left[g_m\frac{e\hbar}{2mb_{mn}}\gamma_{mn}\gamma_{Mn}\,(\boldsymbol{s}_m\cdot\boldsymbol{B}) - g_M\frac{Ze\hbar}{2Mb_{Mn}}\gamma_{mn}\gamma_{Mn}\,(\boldsymbol{s}_M\cdot\boldsymbol{B})\right]. \tag{14}$$

Averaging this operator with the Schrödinger wave function $\psi(\boldsymbol{r})$, due to the normalization requirement,

$$\psi(\boldsymbol{r}) = (b_{mn}b_{Mn}\gamma_{mn}\gamma_{Mn})^{3/2}\,\psi(\boldsymbol{r}'), \tag{15}$$

implied by the transformation (6), we obtain

$$\left(\bar{V}_{\text{PBFT}}\right)_{mag} \equiv (W_b)_{mag} = b_{mn}b_{Mn}{\gamma_{mn}}^2{\gamma_{Mn}}^2\left[g_m\frac{e\hbar b_{Mn}}{2m}\,\left(\overline{\boldsymbol{s}_m\cdot\boldsymbol{B}}\right) - g_M\frac{Ze\hbar b_{mn}}{2M}\,\left(\overline{\boldsymbol{s}_M\cdot\boldsymbol{B}}\right)\right], \tag{16}$$

where $(W_{\text{PBFT}})_{mag}$ gives the Zeeman splitting of energy levels in PBFT framework. Herein in the averaging of $(\bar{V}_{\text{PBFT}})_{mag}$ we can put $\boldsymbol{B}(\boldsymbol{r}) = \text{const}$, which is always fulfilled in the atomic scale.

Inserting eqs. (2a)–(2d) into eq. (16), we obtain for the nS-state

$$(W_{\text{PBFT}})_{mag} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2M_Zm_{\text{Z}}}{(m_{\text{Z}}+M_{\text{Z}})^2}\right)\left[W_{mag} - \frac{(Z\alpha)^2}{n^2}\frac{e\hbar}{2\,(M_{\text{Z}}+m_{\text{Z}})}\left(\overline{(g_m\boldsymbol{s}_m - Zg_M\boldsymbol{s}_M)\cdot\boldsymbol{B}}\right)\right], \tag{17}$$

(with the sufficient accuracy $(Z\alpha)^2$), where W_{mag} stands for the Zeeman splitting of energy levels, obtained via the averaging of common operator (13). Here we supply the masses m_{Z}, M_{Z} by the subscript “Z” (“Zeeman effect”), in order to distinguish them from the masses m, M in eqs. (9)–(12), for the atom constituents used in the measurement of spin-spin splitting.

For the sublevel $F = 1$ used for the measurement of Zeeman effect, $\boldsymbol{s}_m - \boldsymbol{s}_M = 0$, and eq. (17) reads

$$(W_{\mathrm{PBFT}})_{mag} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2m_{\mathrm{Z}}M_{\mathrm{Z}}}{(m_{\mathrm{Z}}+M_{\mathrm{Z}})^2}\right)\left(W_{mag} - \frac{(Z\alpha)^2}{n^2}\frac{(g_m - Zg_M)e\hbar}{2(M_{\mathrm{Z}}+m_{\mathrm{Z}})}\left(\overline{\boldsymbol{s}_m \cdot \boldsymbol{B}}\right)\right). \tag{18}$$

Now we remind that the magnetic moments for the constituents of hydrogenlike atoms are determined experimentally via the measurement of Zeeman effect in these atoms, and the PBFT correction (18) to this effect stipulates the corresponding corrections to the magnetic moments for bound particles. One can see that such corrections, being accounted in the expression for the common hyperfine spin-spin splitting W_{s-s} of eq. (12b), induce a decrease the total PBFT correction in comparison with the factor $(1 - \frac{(Z\alpha)^2}{n^2}\frac{2mM}{(M+m)^2})$ appeared in eq. (12b).

Having obtained eqs. (12b) and (18), we are now in the position to analyze HFS in various hydrogenlike atoms with the derivation, when necessary, the total PBFT correction, taking into account the correction to the Zeeman effect and related correction to magnetic moments of bound particles.

3.1 Hydrogen

For the ratio of electron to proton masses $m_e/M_p \approx 1836$, the correction of eq. (12b),

$$\delta\left(W_{\mathrm{PBFT}}\right)_{s-s} = -\frac{(Z\alpha)^2}{n^2}\frac{2m_eM_p}{(m_e+M_p)^2}W_{s-s}, \tag{19}$$

occurs much less than the present calculation uncertainty of HFS in hydrogen. Indeed, for the $1S$ state $W_{s-s} = 1\,420\,405.751\,768(1)\,\mathrm{kHz}$ [11], and according to eq. (19) at $n = 1$,

$$\delta\left(W_{\mathrm{PBFT}}\right)_{s-s} \approx 80\,\mathrm{Hz}, \tag{19a}$$

while the nuclear structure contribution varies from tens to hundreds kHz [12–14]. Correspondingly, one can expect that the correction (19a) does not alter the value of the proton Zemach radius evaluated via spin-spin interaction in PBFT framework.

Let us demonstrate the validity of this assumption via the direct calculations, using the known expression [15,16] for HFS, which includes the Zemach radius R_{Z},

$$W_{\mathrm{HFS}} = E^{\mathrm{F}}\left(1 + \delta^{\mathrm{QED}} + \delta^{\mathrm{recoil}} + \delta^{\mathrm{pol}} + \delta^{\mathrm{hvp}} + \delta^{\mathrm{weak}} - 2\alpha m_{ep}R_{\mathrm{Z}}\right) \tag{20}$$

(in the units $\hbar = c = 1$). Here E^{F} is the Fermi energy, δ^{QED} is the correction due to anomalous magnetic moment of electron and the high-order (α^4 and higher) QED contributions, δ^{recoil} collects the contribution of all terms which depend on the ratio m/M, δ^{pol} in the proton polarizability correction, δ^{hvp} describes the strong interaction effects outside the proton, δ^{weak} is the weak interaction term, and

$$m_{ep} = m_eM_p/\left(m_e + M_p\right) \tag{21}$$

is the reduced mass of the bound electron ($m = m_e$) and proton ($M = M_p$).

In practice the Zemach radius is evaluated by the comparison of experimental data for the hyperfine spin-spin interval in hydrogen with theoretical value (20) and its modern value is equal to [16]

$$R_{\mathrm{Z}} = 1.045(16)\,\mathrm{fm}. \tag{22}$$

Now we want to re-evaluate the Zemach radius in the framework of PBFT, which results from the PBFT correction to spin-spin interaction (eq. (12b)).

Combining eqs. (20) and (12b) for the case of hydrogen, we obtain

$$(W_{\mathrm{PBFT}})^{\mathrm{H}}_{\mathrm{HFS}} = \left(1 - \frac{2m_eM_p\alpha^2}{(M_p+m_e)^2}\right)E^{\mathrm{F}}\left(1 + \delta^{\mathrm{QED}} + \delta^{\mathrm{recoil}} + \delta^{\mathrm{pol}} + \delta^{\mathrm{hvp}} + \delta^{\mathrm{weak}} - 2\alpha m_{ep}(R_{\mathrm{Z}})_{\mathrm{PBFT}}\right). \tag{23}$$

Using eqs. (20), (21) and (23), we determine the difference between the values $(R_{\mathrm{Z}})_{\mathrm{PBFT}}$ and R_{Z}, which gives the change of Zemach radius in PBFT in comparison with the common value (22)

$$\begin{aligned}\delta(R_{\mathrm{Z}})^{\mathrm{H}}_{\mathrm{PBFT}} &= (R_{\mathrm{Z}})_{\mathrm{PBFT}} - R_{\mathrm{Z}} = -\frac{1}{2\alpha m_{ep}}\frac{W_{\mathrm{HFS}}}{E^{\mathrm{F}}}\left[\left(1 - \frac{2m_eM_p\alpha^2}{(M_p+m_e)^2}\right) - 1\right]\\ &= \frac{1}{m_{ep}}\frac{m_eM_p\alpha}{(M_p+m_e)^2}\frac{W_{\mathrm{HFS}}}{E^{\mathrm{F}}} \approx \frac{\alpha}{M_p+m_e}.\end{aligned}$$

Here we put $W_{\rm HFS}/E^{\rm F} = 1$, which is a sufficient approximation in the estimation of $\delta(R_{\rm Z})_{\rm PBFT}$. In usual units the latter expression reads

$$\delta(R_{\rm Z})^{\rm H}_{\rm PBFT} = \frac{\hbar\alpha^2}{(M_p + m_e)c}\,. \tag{24}$$

After the substitution of corresponding numerical values in eq. (24) we obtain

$$\delta(R_{\rm Z})^{\rm H}_{\rm PBFT} = 1.1 \cdot 10^{-3}\,\text{fm}. \tag{25}$$

The value (25) is more than one order of magnitude less than the present measurement uncertainly of $R_{\rm Z}$ (see eq. (22)) and can be well ignored.

The obtained result exempts us from any need to introduce into eq. (12b) the PBFT correction to the magnetic moments for both the electron and proton, which stems from the modified eq. (17) for Zeeman effect. As we have mentioned above, this kind of correction can only reduce the value (25).

3.2 Positronium

For positronium $m = M = m_e$, and eq. (12b) takes the form

$$(W_{\rm PBFT})^{\rm Ps}_{s-s} = \left(1 - \frac{(Z\alpha)^2}{2n^2}\right) W^{\rm Ps}_{s-s}. \tag{26}$$

The correction of eq. (26) strongly dominates over the PBFT correction to the magnetic moments of electron (positron), because the latter is determined via the Zeeman effect in atoms, where $m_{\rm Z} \ll M_{\rm Z}$. Thus eq. (26) can be directly involved to the re-estimation of HFS in positronium. In particular, it reduces the $1S$ hyperfine interval from the commonly calculated value $W^{\rm Ps}_{\rm HFS} = 203\,391.7(8)\,\text{MHz}$ [12] to $(W_{\rm PBFT})^{\rm Ps}_{\rm HFS} = 203\,386.0(8)\,\text{MHz}$. This estimation better agrees with the available experimental data 203 389(2) MHz [17] and 203 387(2) MHz [18], than the common calculated result.

3.3 Muonium

For this atom we put in eq. (12b) $m = m_e$, $M = m_\mu$ (the muon mass). Besides, in eq. (18) we replace $g_m \to g_e$, $g_M \to g_\mu$ and put with the high accuracy $g_e = g_\mu$ in the correcting term of the order $(Z\alpha)^2$. In this equation we also take $m_{\rm Z} = m_e$, $M_{\rm Z} = m_\mu$, so that eq. (17) acquires the form

$$(W_{\rm PBFT})^{\rm Mu}_{mag} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2m_e m_\mu}{(m_e + m_\mu)^2}\right) W^{\rm Mu}_{mag}. \tag{27}$$

Comparing now eq. (27) with the PBFT expression for spin-spin interval (12b) in muonium (where we also put $m = m_e$, $M = m_\mu$), we reveal that in both cases (Zeeman effect and HFS), the correction of PBFT is exactly the same.

Now we again remind that the magnetic moment of muon $\boldsymbol{\mu}_\mu$ entering into eq. (12b) for spin-spin interval is extracted via the comparison of calculated and experimental data for the Zeeman splitting in muonium (27)[1]. Since in common theory the PBFT correction $\left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2m_e m_\mu}{(m_e+m_\mu)^2}\right)$ does not appear in eq. (27), we have to conclude that the common approach leads to underestimation of muon magnetic moment, *i.e.*

$$(\mu_\mu)_{\rm c} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2m_e m_\mu}{(m_e + m_\mu)^2}\right)(\mu_\mu)_{\rm PBFT}. \tag{28}$$

The subscript "c" means "common", and here we suppose that PBFT is the correct theory, so that $(\mu_\mu)_{\rm PBFT}$ represents the true value of the magnetic moment for muon. Then, after the substitution of $(\mu_\mu)_{\rm PBFT}$ in eq. (12b), we see that both correcting factors of eqs. (27) and (12b) cancel away, and we arrive at the common expression for spin-spin interval in muonium, which, as known, for the $1S$ state perfectly agrees with experimental data (*e.g.*, [11]).

[1] Here we adopt that PBFT correction to electron's magnetic moment is much smaller than the correction to the muon's magnetic moment, insofar as the former is measured for atoms, where the ratio m/M is much smaller than the ratio m_e/m_μ. In particular, one can show that the PBFT corrections to electron's g-factor and its rest mass do not exceed the present uncertainties in their determination.

However, we point out that in this way the conventional approach has got the false value of magnetic moment $(\mu_\mu)_\mathrm{c}$, as eq. (28) shows. Presenting now the magnetic moment in the explicit form,

$$\boldsymbol{\mu}_\mu = g_\mu e \boldsymbol{s}_\mu / m_\mu,$$

we further take into account that the g-factor for bound muon and its spin are both determined with much better precision that muon mass [11] and hence the uncertainty of their determination can be ignored in further analysis. Thus the deviation of $(\mu_\mu)_\mathrm{c}$ from the true value $(\mu_\mu)_\mathrm{PBFT}$ induces overestimation of muon rest mass, *i.e.*[2]

$$(m_\mu)_\mathrm{c} = (m_\mu)_\mathrm{PBFT} \Big/ \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2m_e m_\mu}{(m_e+m_\mu)^2}\right). \tag{29}$$

Summarizing this analysis, we finally emphasize that for the muonium atom, being considered in PBFT framework, the expressions for spin-spin interval and Zeeman splitting are scaled exactly by the same correcting factor. Since the experiments of both kinds are entangled to each other through the magnetic moment of muon, the related data occur insensitive to the PBFT correction to magnetic moment of muon (28) and to its mass (29). As a result, the spectroscopic data for muonium are not suitable for an experimental test of PBFT, even if this theory predicts the lower value of true muon mass (29) in comparison with its present estimation.

A quite different situation emerges for the muonic hydrogen atom, where the increase of the ratio of m/M by two orders of magnitude in comparison with hydrogen and one order of magnitude in comparison with muonium, makes the specific effects of PBFT to be observable, at least in principle, in the measurement of Zeeman splitting.

3.4 Muonic hydrogen

For this atom we use $m = m_\mu$, $M = M_p$ (the proton mass) for the two-body problem. Thus the factor $\frac{(Z\alpha)^2}{n^2}\frac{2mM}{(m+M)^2}$ entering into PBFT correction of eqs. (12b) and (18), occurs one order of magnitude larger than for muonic hydrogen, and two orders of magnitude larger than for hydrogen.

Nevertheless, in the analysis of spin-spin splitting, such an increased value of PBFT correction does not lead to any measurable deviations from the common results. The reason is a large contribution of nuclear size effect in muonic hydrogen, which masks the corrections of PBFT. In particular, let us show that the Zemach proton radius estimated for muonic hydrogen in the framework of PBFT does not lead to its deviation from the common value (22) within the present uncertainty.

This result can be immediately obtained via combining eqs. (20), (21) and (23) for the case of muonic hydrogen. Hence we obtain

$$\delta(R_\mathrm{Z})^{\mathrm{muonic\,H}}_{\mathrm{PBFT}} = \frac{\hbar\alpha^2}{(M_p+m_\mu)c}. \tag{30}$$

After the substitution of the corresponding numerical values in eq. (30) we get

$$\delta(R_\mathrm{Z})^{\mathrm{muonic\,H}}_{\mathrm{PBFT}} = 1.0\cdot 10^{-3}\,\mathrm{fm}. \tag{31}$$

Thus, like for the case of hydrogen (eq. (25)), the PBFT correction (31) to the Zemach radius is more than one order of magnitude less than its present measurement uncertainly (see eq. (22)) and can be well ignored. Since the PBFT correction to the spin-spin interval scales as n^{-2}, the same conclusion holds true for the $2S$ hyperfine splitting, too.

However, for the Zeeman effect in muonic hydrogen, the PBFT corrections to common calculations are substantial and can be subjected to an experimental test.

First of all, we address to the general equation (18) for PBFT correction to Zeeman effect and observe that now $g_\mu \neq g_p$ ($g_\mu \approx 2.0, g_p \approx 5.6$), so that the correcting term containing the difference $(g_\mu - g_p)$ is not vanishing. However,

[2] The PBFT re-estimation (29) to muon mass forces us to remember that the normalizing coefficient in the wave function used for averaging of operators of spin-spin interaction (11) and Zeeman effect (14), contains the reduced mass of electron and muon $m_R = (\frac{m_e m_\mu}{m_e+m_\mu})$ to the 3/2 power [19]. Hence one can see that in this case eq. (28) should be replaced by the relationship

$$(\mu_\mu)_\mathrm{c}(m_R)^3_\mathrm{c} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2m_e m_\mu}{(m_e+m_\mu)^2}\right)(\mu_\mu)_\mathrm{PBFT}(m_R)_\mathrm{PBFT}{}^3.$$

However, the simple calculations (which are omitted here for brevity) show that the PBFT correction to reduced mass occurs much smaller than the correction to the muon magnetic moment (28) and the related correction to muon mass (29). Thus eqs. (28) and (29) can be left unchanged.

one can easily realize that it is one order of magnitude smaller (as the ratio m_μ/M_p) than the leading correcting term in the expression $\left(1-\frac{(Z\alpha)^2}{n^2}\frac{2m_\mu M_p}{(m_\mu+M_p)^2}\right)W_{mag}$ issuing from the interaction of muon magnetic moment with an external magnetic field, entering into W_{mag}. Thus, in the qualitative estimation of PBFT correction to the Zeeman effect, the last term in rhs of eq. (18) can be neglected, and this equation takes the form

$$(W_{\mathrm{PBFT}})_{mag} = \left(1-\frac{(Z\alpha)^2}{n^2}\frac{2m_\mu M_p}{(m_\mu+M_p)^2}\right)W_{mag}. \tag{32}$$

Now we recall that the energy W_{mag} contains the terms to be proportional to the magnetic moment of muon $(\boldsymbol{\mu}_\mu)_{\mathrm{c}}$ and proton $(\boldsymbol{\mu}_p)_{\mathrm{c}}$, and the PBFT correction to $(\boldsymbol{\mu}_\mu)_{\mathrm{c}}$ determined by eq. (28) dominates over the correction to $(\boldsymbol{\mu}_p)_{\mathrm{c}}$. What is more, for the muonic hydrogen atom the correction (28) can be ignored in comparison with the correcting factor of eq. (32). Hence we come to conclude that eq. (32) can be taken for the determination of the total PBFT correction to the Zeeman effect in muonic hydrogen at least in the qualitative analysis.

For the $1S$ state and for the ratio $m_\mu/M_p \approx 207/1836$, the factor

$$\frac{(Z\alpha)^2}{n^2}\frac{2m_\mu M_p}{(m_\mu+M_p)^2} \approx 0.95\cdot 10^{-5} \tag{33}$$

characterizes the relative deviation of the result expected in PBFT framework from the common calculations of Zeeman splitting implemented with the Breit-Rabi expression [10]

$$W_m^{\pm} = -\frac{W_{s-s}}{4} - g_\mu \mu_p m B \pm \frac{W_{s-s}}{2}\sqrt{1+4mx+x^2}. \tag{34}$$

Here B stands for the external magnetic field,

$$x = \frac{(g_{\mu J} + g_\mu\,(\mu_p/\mu_\mu))\,\mu_\mu B}{W_{s-s}},$$
$$g_{\mu J} = g_L\frac{j(j+1)-s(s+1)+l(l+1)}{2j(j+1)} + g_L\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)},$$

$\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$, where $\boldsymbol{l}$ is the orbital angular momentum, and $\boldsymbol{s}$ is the spin.

One can expect the measuring uncertainly for the Zeeman splitting of $1S$ level about 1 ppm, which makes the correction (33) to be observable in NMR measurements for muonic hydrogen in the presence of external magnetic field.

In order to estimate the absolute value of deviation between Zeeman effect of $1S$ state in common theory and PBFT, we use the results of calculations of [20], which on the basis of eq. (34) give the value of the Zeeman splitting for $2S_{1/2}$-$2P_{3/2}$ transition ($m_F = +1, -1$) about 20 MHz for $B = 5$ T. Adopting approximately the same splitting for the $1S$ state (which corresponds to the condition $x \ll 1$ in eq. (34)), and taking the external magnetic field strength about 50 T (which can be achieved in the pulse mode), we get the value of Zeeman splitting about

$$\Delta W_{\mathrm{Z}} \approx 200\,\mathrm{MHz}. \tag{35}$$

Thus, in a view of eq. (33), the deviation between PBFT and common result becomes

$$\delta W = -\frac{(Z\alpha)^2}{n^2}\frac{2m_\mu M_p}{(m_\mu+M_p)^2}\Delta W_{\mathrm{Zeeman}} \approx -2\,\mathrm{kHz}. \tag{36}$$

Now we remind that the accuracy of measurement of HFS in muonium is about 50 Hz, which is much less than the value (36). Assuming that the same uncertainly can be achieved in the measurement of Zeeman effect in $1S$ state of muonic hydrogen, we come to conclude that the value (36) should be observable in modern experiments.

3.5 PBFT re-estimation of muon mass

Thus, if the decrease of Zeeman splitting in the muonic hydrogen atom by about 2 kHz in comparison with commonly expected value will be detected, as eq. (36) predicts, this result would simultaneously mean that the muon mass should be re-estimated according to eq. (29). Numerically the correcting factor in eq. (29) $\frac{(Z\alpha)^2}{n^2}\frac{2m_e m_\mu}{(m_e+m_\mu)^2}$ for the $1S$ state is equal to $2.552\cdot 10^{-7}$. Therefore, using the present CODATA value for the muon mass [21]

$$(m_\mu)_{\mathrm{CODATA}} = 206.768\,2843(52)$$

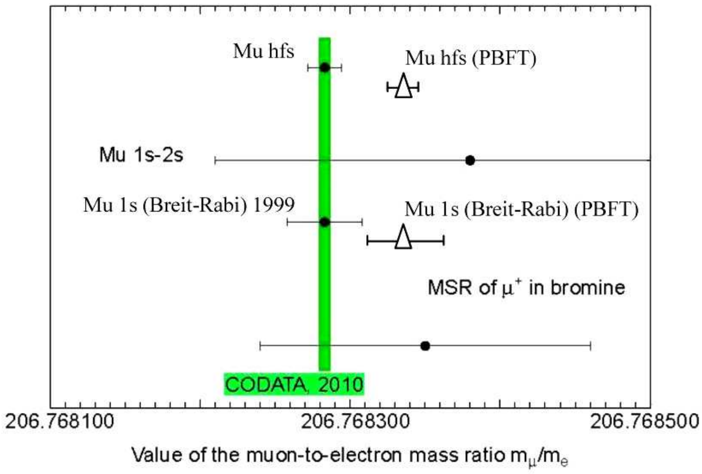

Fig. 1. Reproduced from ref. [11], where we indicated the updated CODATA value for muon mass. The PBFT re-estimations of muon mass for Mu hfs experiment [22] and Mu 1s Breit-Rabi experiment [22] are indicated by triangles.

(expressed in the units of electron's mass), we derive the new (according to PBFT, true) value of the muon mass

$$(m_\mu)_{\mathrm{PBFT}} = 206.768\,3371(52).$$

One can see that the difference between $(m_\mu)_{\mathrm{PBFT}}$ and $(m_\mu)_{\mathrm{CODATA}}$ is one order of magnitude larger than the uncertainty in determination of muon mass presented by CODATA. However, here one should emphasize that this uncertainty is mainly determined by the accuracy of HFS experiment in muonium [22], which is much smaller than the uncertainty of other known experiments for the measurement of m_μ [23,24]. At the same time, we have shown above that the PBFT correction to muon mass (29) occurs unaccounted in HFS experiment in muonium. It means that, according to PBFT, the muon mass to be found in this experiment should be divided by the correcting factor of eq. (29). This circumstance becomes principal in the comparative analysis of different experiments for the determination of muon mass.

In fig. 1 we reproduce the plot from ref. [11], where the results of measurement of m_μ in various experiments are collected. We complement this plot by the present CODATA value of m_μ, as well as by the PBFT re-estimation of muon mass on the basis of eq. (29) for HFS muonium experiment (Mu hfs [20]), and for the latest Zeeman effect experiments (Breit-Rabi, 1999 [22]), indicated by triangles. We see that these re-estimated values are much closer to the mean value of m_μ obtained in the measurement of $1S$–$2S$ interval in muonium (Mu 1s–2s) [23] and in muon-spin-precession–resonance (MSR) experiment [24]. A large uncertainty of Mu 1s–2s experiment [23] allows us to exclude it from further analysis. Concerning MRS experiment [24], we especially emphasize that it is free from any PBFT corrections and thus it can be classified as the basic experiment, which directly yields the correct muon mass.

At the moment, a comparably large uncertainty of the result of MRS experiment [24] does not allow making a crucial choice either in the favor of PBFT, or against PBFT. We see that both the common results and the PBFT prediction with respect to the muon mass lie within the range of uncertainty of this experiment, though the PBFT re-estimations are substantially closer to the mean value of muon mass extracted from MRS measurements, than the common results. Thus, new high-precision MRS measurements are required for the crucial test of PBFT, aimed for the determination of true muon mass.

4 Conclusion

In the present contribution we have analyzed hyperfine spin-spin splitting (HFS) in various hydrogenlike atoms and suggested two new independent tests of PBFT, which we had developed earlier. One of them is based on the measurement of Zeeman effect of $1S$ level of muonic hydrogen in a strong (about 50 T) magnetic field. The predicted deviation between common result and PBFT calculation is about 2 kHz (or 10 ppm in relative units), which can be measured by NMR method, where the measurement uncertainly should be at least one order of magnitude smaller. Another independent test of PBFT can be done in the repetition of the MRS experiment with an enhanced precision in comparison with the known experiment [24]. Unlike the HFS and Zeeman experiments, the results of MRS measurements with respect to estimation of muon mass do not imply any PBFT corrections and thus, if the precision of this experiment will be improved, it will be capable to distinguish the difference in predictions of muon mass derived in the common approach and in the framework of PBFT.

We emphasize that the proposed new experimental test of PBFT (which presently gives the amazing coincidence between calculated and measured data in precise physics of simple atoms [2]) is not only significant from a general viewpoint, but seems to be very topical in the resolution of the available remarkable disagreement between spectroscopic data for muonic hydrogen [7] and scattering experiment [8,9] in the estimation of proton charge radius. Thus we believe that the realization of the experiments proposed in the present paper, will shed light on the resolution of the proton-size puzzle, too.

Finally, we notice that the PBFT corrections for hyperfine spin-spin splitting (HFS) in four different hydrogenlike atoms analyzed in this paper show remarkable property —they do not destroy an agreement between theory and experiment, where such agreement already exists and improve the coincidence with experiment where it is on due. In particular for the proton dimensional parameters PBFT predicts the reduced value of proton charge radius by about 4% in comparison with the modern CODATA value [2], but leaves the proton Zemach radius practically unchanged (see eqs. (25) and (31)) in comparison with the currently adopted value (22). The latter result can be also subjected to the experimental test in spectroscopy of the muonic hydrogen atom [6].

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