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POLEMICS

UNIFICATION OF SPACE-TIME-MATTER-ENERGY*

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Abstract. A complete description of space-time, matter and energy is given in Einstein's special theory of relativity. We derive explicit equations of motion for two falling bodies, based upon the principle that each body must subtract the mass-equivalent for any change in its kinetic energy that is incurred during the fall. We find that there are no singularities and consequently no blackholes.

Keywords: binding energy, elementary forces, energy conservation, equivalence principle, Minkowski spacetime, rest-mass, special relativity, two body problem.

AMS Subject Classification: 83A05, 83D05, 83C57, 81V22.

Introduction

Special Relativity has proven itself to be an exceptionally powerful theory that has revolutionized human understanding of the material universe in the 20th Century [4], [11]. The purpose of the present article is to show how by imposing a strict *local conservation of energy* in the special theory of relativity, the theory takes on a new elegance and universality.

Einstein, stating that all inertial frames are equally valid, based his special theory of relativity on the *principle of relativity* of Galileo [8], [9], [10], [13], [15]. On the other hand, Einstein's *Principle of Equivalence* identifying "gravitational mass" with "inertial mass", upon which he based his general theory of relativity, asserts that the effects of gravity and acceleration are indistinguishable [7]. The principle of equivalence between acceleration and gravitation leads to inconsistencies in the breaking of the laws of conservation of energy and momentum [25], [26], and the breaking of the mass-energy equivalence expressed in the famous formula $E = mc^2$, which is one of the pillars of the special theory of relativity [27], [28], [29]. An interesting discussion of the origin of this famous formula can be found in [2].

There are other objections that can be raised to the general theory of relativity, particularly in regards to the existence of singularities [31]. Both Yilmaz and Logunov have proposed exponential metrics (yielding no black holes) [24], [46], instead of the Schwarzschild metric [32], [33]. Yilmaz went further to question whether in Einstein's general theory Newton's apple would actually fall [48]. Other authors have found that the *principle of equivalence of gravitational mass and inertial mass* in general relativity is problematical for different reasons [22], [23], [24], [44]. Indeed, it is shown in [46], [47], [48], that Einstein's field equations are not satisfied in an accelerated elevator. Also, see the references [1], [5], [6] and [17].

* The subject of this paper does not correspond the profile of our journal. Nevertheless, taking into account the importance of general physical theories discussed in the paper, the Editorial Board decided to publish it and hopes that the paper could initiate the discussion of our readers.

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Recently, a team led by A. Kholmetskii at Belarusian State University performed a Mossbauer experiment to see how a nuclear clock mounted at the edge of a rotor is affected by rotational motion. They were able to verify, with a high degree of precision, a prediction made by Yarman et al. [17]. They found, contrary to the prediction made by Einstein [5], that the clock is not only affected by its tangential velocity, but also by its binding to the acceleration field. The overall time dilation is practically twice as much as predicted classically. These results, which will soon appear, challenge the validity of the Principle of Equivalence in General Relativity, and favor the predictions of our theory.

In [38], the second author considered how a single object would fall in the gravitational field of a celestial object infinitely more massive. In what follows, we derive the exact equations of motion for two bodies of arbitrary masses under the influence of any of the known elementary forces in nature. In addition, we consider a simplified three body problem. We find that there are no singularities, even in the case of point-like masses. Surprisingly, despite the inverse square dependency of Newton gravitational attractive force, there still appears no singularity at $r = 0$, owing to the fact that the smaller mass self-annihilates at exactly the point where one would expect the singularity to occur. The two essential ingredients of our approach, in the framework of Einstein's *special theory of relativity*, are the *principle of local conservation of energy-momentum* and the famous mass energy relationship $E = mc^2$.

Section 1, defines the concept of rest-mass utilized in our theory. Whereas Einstein, by his *equivalence principle*, considers "inertial mass" and "rest-mass" to be equivalent, we believe that there is a clear asymmetry between an accelerating elevator and a gravitational field. An observer must get *accelerated* to be able to catch up with an accelerating elevator, whereas he has to get *decelerated* in order to be able to land on the celestial body. In our theory, the first process yields a *mass increase*, whereas the second one leads to a *mass decrease* [43]. It follows that the idea that the rest-mass of an object is a fundamental constant of nature, must be replaced by the concept of the *instantaneous rest-mass* of an object in a non-homogeneous field, as was first done in [38].

Section 2, defines the concept of *binding energy* of a two body system to account for the work done by any one or all of the four fundamental forces of nature. We express the ideas of special relativity in the framework of the *spacetime algebra* (STA) of 4-dimensional Minkowski space developed by D. Hestenes [18]. In STA, each relative frame of an observer is defined by a unique, future pointing, Minkowski timelike unit vector tangent to the timelike curve called the *history* of that observer. The exact relationship between STA and the 3-dimensional Euclidean space of the World of experience has been further explored in [36]. The rich structure of lower dimensional Minkowski spaces has recently been studied in [16] and [37].

Section 3, calculates the change of mass for each mass in a closed two body system as a function of the *total binding energy* of the two bodies, as they move under whatever the forces of nature. We find explicit formulas both for the masses and also for the velocities of the two masses. All our calculations are based upon the simple principle that each body, as it moves under the forces of nature, must subtract the mass-equivalent for any change in its kinetic energy.

Section 4, considers the binding energy due to Newton's gravitational force between two bodies. We derive explicit solutions where possible, and a numerical solution for the cases when this is not possible. We also consider the simplified three body problem on a straight line and where two of the bodies have the same mass. It is surprising that the inverse square dependency of both Newton's law for gravitational attraction, and the Coulomb force law, can be derived as a requirement imposed by the special theory of relativity [38], [43].

In the final Section 5, we discuss the relationship and generalization of our theory to include quantum mechanics, based upon previous work that has been done by the second author. As a consequence of the explicit solutions to the two body problem, which we have found in section 4, we deduce that black holes with a well defined Schwarzschild radius cannot exist.

1. The concept of rest-mass

We begin by defining the *rest-mass* m_∞ of a body to be the mass of the body when it is at an infinite distance away from all other bodies and forces in the Universe, as measured by an observer traveling at relative rest with respect to that body. The great advantage of the STA of Hestenes, for the most part still unappreciated by the physics community, is that each such inertial frame is uniquely characterized by a constant Minkowski time-like unit vector u. See [18] and [21] for details of the spactime algebra formulation of special relativity which we use throughout this paper.

Let p_∞ be the *Minkowski energy-momentum vector* of the rest-mass m_∞. Since we have assumed that m_∞ is at rest in the frame defined by u, it follows that $p_\infty = m_\infty c^2 u$. Now let $v = \frac{dx}{d\tau}$ be the Minkowski timelike unit vector of an observer with the timelike history $x = x(\tau)$, where τ is the natural parameter of *proper time* (arc length). The unit vector $v = v(\tau)$ uniquely defines the *instantaneous frame* of the observer at the proper time τ.

As measured from the rest-frame u to the instantaneous relative frame v, we have

$$p_\infty v = m_\infty c^2 uv = m_\infty c^2 (u \cdot v + u \wedge v) = \gamma_v m_\infty c^2 (1 + \frac{\mathbf{v}}{c}), \tag{1}$$

where $\gamma_v = u \cdot v = \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}}$ and $\frac{\mathbf{v}}{c} = \frac{u\wedge v}{u\cdot v}$. We say that $E_v = p \cdot v = \gamma_v m_\infty c^2$ is the instantaneous *relative energy*, $\mathbf{p}_v = \gamma_v m_\infty c^2 \frac{\mathbf{v}}{c}$ is the instantaneous *relative momentum*, and $\mathbf{v}$ is the instantaneous *relative velocity* of m_∞ in the instantaneous frame v as measured by u. This convention is opposite by a sign to the convention used by Hestenes in his 1974 paper. We use the same convention here as was used by Sobczyk in [37]. There are many different languages and offshoots of languages that have been used to formulate the ideas of special relativity. For a discussion of these and related issues, see [3], [34], [35]. A unified language for mathematics and physics has been proposed in [19].

Equation (1) shows that with respect to the relative frame v, the mass m_∞ has the increased relative energy $E_v = \gamma_v m_\infty c^2$. This means that if we want to boost the mass m_∞ from the rest-frame u into the instantaneous frame v, we must expend the energy $\triangle E_1 = (\gamma_v - 1)m_\infty c^2$ to get the job done. Expanding the right-hand side of this last equation in a Taylor series in $|\mathbf{v}|$, we find that

$$\Delta E_1 = \frac{m_\infty}{2}\mathbf{v}^2 + \frac{3m_\infty}{8c^2}\mathbf{v}^4 + \frac{5m_\infty}{16c^4}\mathbf{v}^6 + \cdots . \tag{2}$$

For velocities $|\mathbf{v}| << c$, we see that the energy expended to boost the mass m_∞ into the instantaneous frame v moving with velocity $\mathbf{v}$ with respect to the rest-frame u is $\Delta E_1 \cong \frac{m_\infty}{2}\mathbf{v}^2$, which is the classical Newtonian expression for *kinetic energy* of the mass m_∞ moving with velocity $|\mathbf{v}|$.

If, instead, we pay for the work done by deducting the required energy-equivalent from the mass m_∞, to get the residual rest-mass $m = \frac{m_\infty}{\gamma_v}$, then the terminal energy-momentum vector of the mass m_∞ when it has reached the velocity $\mathbf{v}$ is

$$p = mc^2 v = \frac{m_\infty}{\gamma_v}c^2 v = \frac{p_\infty}{\gamma_v}uv = e^{-\frac{\phi\hat{\mathbf{v}}}{2}}\frac{1}{\gamma_v}p_\infty e^{\frac{\phi\hat{\mathbf{v}}}{2}}. \tag{3}$$

In this equation, $\hat{\mathbf{v}}$ is a unit relative vector in the direction of the velocity $\mathbf{v}$, and $c\tanh(\phi) = |\mathbf{v}|$ is the magnitude of the velocity as measured in the rest-frame u.

Equation (3) has some easy but important consequences. We first note that $m = \frac{m_\infty}{\gamma_v} = 0$ when $|\mathbf{v}| \to c$. This means that the energy content of each material body is exactly the energy which would be required to accelerate the body to the speed of light c. Assuming that we have a one hundred percent efficient photon drive, the body would reach the speed of light at precisely the moment when its last bit of mass-equivalent is expelled as a photon. A second interesting observation is that when we expand $(m_\infty - m)c^2 = m_\infty(1 - \frac{1}{\gamma_v})c^2$ in a Taylor series in $|\mathbf{v}|$ around

$|\mathbf{v}| = 0$, we obtain

$$\Delta E_2 = (m_\infty - m)c^2 = \frac{m_\infty}{2}\mathbf{v}^2 + \frac{m_\infty}{8c^2}\mathbf{v}^4 + \frac{m_\infty}{16c^4}\mathbf{v}^6 + \cdots = \frac{\Delta E_1}{\gamma_v}. \tag{4}$$

Whereas the expressions $\Delta E_1 \cong \Delta E_2$ for $|\mathbf{v}| << c$, the expression for ΔE_2 is much closer to the classical kinetic energy over a much larger range of velocities $|\mathbf{v}| < c$, and differs only by a factor of 2 when $|\mathbf{v}| = c$.

The basic premise upon which our theory is built is that when any particle evolves on its *timelike curve* $x(\tau)$, subjected only to the elementary forces of nature and satisfying the initial condition that $p(0) = m_\infty c^2 u$, then its energy-momentum vector has the form $p(\tau) = m(\tau)c^2 v(\tau)$ for $m(\tau) = \frac{m_\infty}{\gamma_v}$, and satisfies the *conservation law*

$$p(\tau) \cdot u = m_\infty c^2 = constant \tag{5}$$

for all values $\tau \geq 0$. This law is a direct consequence of the local conservation of energy requirement (3). We say that

$$m(\tau) = \frac{\sqrt{p^2}}{c^2} = \frac{p(\tau) \cdot v(\tau)}{c^2} = \frac{m_\infty}{\gamma_v} \tag{6}$$

is the *instantaneous rest-mass* of m_∞ in the instantaneous frame $v(\tau)$.

At the atomic level, our insistance upon the strict local conservation of the *total energy* of each particle (5), means that whenever an elementary particle undergoes a change in its kinetic energy, it must pay for it with a corresponding change in its instantaneous rest-mass (6). Thus, we do not accept that the rest-mass m_∞ of an isolated particle is an invariant when that particle undergoes interactions. Insisting upon a local conservation of energy-momentum has the singular advantage of being completely compatible with the requirements of quantum mechanics, as has been explained elsewhere by the second author [43]. We consider that the *field* of an elementary particle carries only *information* about the *location* of that elementary particle, but does not magically transfer energy across spacetime to affect other elementary particles. Each elementary particle pays for any change in its kinetic energy as it navigates in space, guided by the information supplied by the four elementary forces of Nature. Consequently, an elementary particle annihilates if and only if it reaches the speed of light.

A beautiful discussion and derivation of the basic relationships of relativistic particle dynamics is given in [20] and [21], so we need not rederive them here. We will need, however, a number of special formulas regarding the evolution of a particle whose the energy-momentum vector is given by $p(\tau) = m(\tau)c^2 v(\tau)$ and satisfies (5), as given above. The *Minkowski force* on such a particle as it moves along its timelike curve $x(\tau)$, is given by $f(\tau) = \frac{dp(\tau)}{d\tau}$. It is very easy to calculate the *relative force* $\mathbf{F}(\tau) = \frac{1}{c^2} u f(\tau)$ as measured in the rest frame u. We find that

$$\mathbf{F}(\tau) = \frac{1}{c^2} u f(\tau) = \frac{1}{c^2}\frac{dup(\tau)}{d\tau} = \frac{dt}{d\tau}\frac{d}{dt}(m_\infty + m_\infty \mathbf{v}) = \gamma_v m_\infty \mathbf{a}, \tag{7}$$

where $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ is the *relative acceleration* experienced by the particle as measured in the rest frame u.

Formula (7) is immediately recognized as the relativistic form of *Newton's Second Law.* This form of Newton's Second Law applies to particles subjected only to elementary forces. Noting that $\frac{1}{\gamma_v^2} = 1 - \frac{\mathbf{v}^2}{c^2}$, so that $\frac{d}{dt}(\gamma_v^{-2}) = -2\frac{\mathbf{v} \cdot \mathbf{a}}{c^2}$, it is easy to calculate the useful formulas

$$\frac{d\gamma_v}{dt} = \gamma_v^3 \frac{\mathbf{v} \cdot \mathbf{a}}{c^2} \tag{8}$$

and, with the help of (7),

$$\frac{dm(\tau)}{d\tau} = \gamma_v \frac{dm(\tau)}{dt} = -\gamma_v^2 m_\infty \frac{\mathbf{v} \cdot \mathbf{a}}{c^2} = -\frac{\gamma_v}{c^2}\mathbf{F} \cdot \mathbf{v}, \tag{9}$$

or

$$\frac{dm(\tau)}{dt} = -\frac{1}{c^2}\mathbf{F}\cdot\mathbf{v}. \tag{10}$$

It is well-known that the *total energy-momentum vector* of an isolated n-particle system is a constant of motion in every inertial system [20, p. 634]. Assuming that the only interactions between the particles are the elementary forces, so that (5) applies, it follows that the energy-momentum vector of each particle has the form $p_i(t) = m_i(t)c^2 v_i(t)$, and $p_i(0) = m_i^\infty c^2 u$, where t is the parameter of relative time in the rest-frame u. Assuming further that there are no collisions, this conservation law takes the form

$$P(t) = \sum_{i=1}^{n} p_i(t) = P_0 = \sum_{i=1}^{n} p_i(0) \tag{11}$$

for all $t \geq 0$. Dotting and wedging each side of this equation on the left by u, gives the equivalent statements that

$$u \cdot P(t) = \sum_{i=1}^{n} m_i^\infty c^2 = u \cdot P_0,$$

meaning that the *total energy* of the isolated system is constant, and that the *total linear momentum*

$$u \wedge P(t) = \sum_{i=1}^{n} m_i^\infty c^2 \mathbf{v}_i(t) = u \wedge P_0 = 0$$

of the isolated system is 0 for all values of $t \geq 0$.

2. Two body system

Let us consider an isolated system of two objects $m_i(r)$, with the respective energy-momentum vectors $p_i(r) = m_i(r)c^2 v_i(r)$, for $i = 1, 2$, when they are a distance r from each other as measured in the rest-frame u. This means that the objects can only interact with each other, and that they begin at rest in the rest-frame u when $r = \infty$. Thus, $\lim_{r\to\infty} p_i(r) = m_i^\infty c^2 u$ for $i = 1, 2$.

Current knowledge tells us that there are four fundamental forces in Nature acting between the two objects:

1. *The strong force operating in the nucleus of an atom.* The strong force has a range of about 10^{-15} meters, the diameter of a medium sized nucleus.
2. *The electromagnetic force acting between charged particles.* If two bodies have electric charges q_1 and q_2, respectively, they will experience a *Coulomb Force*

$$F = \frac{kq_1q_2}{r^2}$$

as measured in the rest-frame u, and where $k = 8.98 \times 10^9\, N\, \frac{m^2}{C^2}$ is Coulomb's constant. The Coulomb Force can be attractive or repulsive and is an inverse square relation.
3. *The weak force operating within nuclear particles.* The Weak Force has a range of approximately 10^{-18} meters which is about 1/1000 the diameter of a proton.
4. *Newton's law of gravitational attraction:* The two bodies $m_1(r)$ and $m_2(r)$, in their respective instantaneous frames $v_1(r)$ and $v_2(r)$ at a distance of r, will experience a mutually attractive force

$$F = \frac{Gm_1(r)m_2(r)}{r^2}, \tag{12}$$

where $G = 6.67 \times 10^{-11} N\, \frac{m^2}{kg^2}$ is Newton's constant. We assume that all measurements are carried out in the rest-frame u. Just as for the Coulomb force, the gravitational force obeys an inverse square relation.

Note that our theory requires that both the Newton and Coulomb force laws act between the instantaneous rest-masses, and charges, in their respective instantaneous frames defined by unit Minkowski timelike vectors $v_i(r)$ [38], [43]. Of course, the same modification must be made when applying the weak and strong forces, but we will not consider these forces here.

The conservation law (5) and the conservation law of total energy-momentum (11), applied to our two particle system gives

$$P^\infty = p_1^\infty + p_2^\infty = p_1(r) + p_2(r) = P(r) \tag{13}$$

for all values of $r \geq 0$. Equivalently,

$$u \cdot P^\infty = (m_1^\infty + m_2^\infty)c^2 = u \cdot P(r),$$

which is the conservation of the total energy of the system for all $r \geq 0$, and

$$0 = \frac{u \wedge P^\infty}{c^2} = \frac{u \wedge P(r)}{c^2} = m_1^\infty \mathbf{v}_1(r) + m_2^\infty \mathbf{v}_2(r), \tag{14}$$

which is the conservation of the total linear momentum of the system for all $r \geq 0$.

The quantities

$$E_i^b(r) = p_i(r) \cdot (u - v_i(r)) = m_i^\infty c^2 (1 - \frac{1}{\gamma_i}), \tag{15}$$

which are seen in (4) to be closely related to the classical kinetic energy, are called (by the first author) *Tolga's binding energies* of the respective bodies $m_i(r)$ when they are brought *quasi-statically* (very slowly) to a distance r from each other in the rest-frame u. The *total binding energy* $E^b(r) = E_1^b(r) + E_2^b(r)$, is the work done by the gravitational attraction, or any of the other known three basic forces, acting between the two bodies. With the help of formula (10), we can easily calculate

$$\frac{dE^b}{dt} = -c^2 (\frac{dm_1(\tau_1)}{dt} + \frac{dm_2(\tau_2)}{dt}) = \mathbf{F}_1 \cdot \mathbf{v}_1 + \mathbf{F}_2 \cdot \mathbf{v}_2 = \frac{dE^b}{dr}\frac{dr}{dt}. \tag{16}$$

Whereas we are only interested here in the binding energies of the two bodies due to the force of gravity, or possibly Coulomb forces, all our considerations apply much more broadly [43].

3. Change of mass due to binding energy

Let us directly calculate the change of the rest-masses m_1^∞ and m_2^∞ as the two masses move under the force of gravity or the combined actions of any of the other forces in nature. Very simply, the instantaneous rest-masses $m_i(E_i^b)$ are specified by

$$m_i(E_i^b) = m_i^\infty - \frac{E_i^b}{c^2}, \tag{17}$$

where E_i^b is the instantaneous binding energy of m_i^∞, as follows directly from the binding condition (15). The total binding energy between the instantaneous rest-masses $m_1(E_1^b)$ and $m_2(E_2^b)$ is given by $E^b = E_1^b + E_2^b$. For our considerations below, we will assume that $m_2^\infty = s m_1^\infty$ for a constant value of $s \geq 1$, so that $m_2^\infty \geq m_1^\infty$.

Because of the total binding energy E^b expended by the forces acting between them, as measured in the rest-frame u, the bodies will have gained the respective velocities $\mathbf{v}_1(E^b)$ and $\mathbf{v}_2(E^b)$, fueled by the respective losses to their rest-masses m_1^∞ and m_2^∞. Precisely, we can say that

$$m_i^\infty - f_i \frac{E^b}{c^2} = \frac{m_i^\infty}{\gamma_i} \tag{18}$$

where f_i is the fraction of the total binding energy E^b given up by m_i^∞ for $i = 1, 2$, respectively. This means that $f_1 + f_2 = 1$, and, by the conservation of linear momentum (14), we also know that $(m_1^\infty)^2 \mathbf{v}_1^2 = (m_2^\infty)^2 \mathbf{v}_2^2$ or $\mathbf{v}_2^2 = \frac{1}{s^2}\mathbf{v}_1^2$. Using this information, leads to the system of equations

$$m_1^\infty\Big(1 - \sqrt{1 - \frac{\mathbf{v}_1^2}{c^2}}\Big) - f_1\frac{E^b}{c^2} = 0 \quad \text{and} \quad m_1^\infty\Big(s - \sqrt{s^2 - \frac{\mathbf{v}_1^2}{c^2}}\Big) - (1 - f_1)\frac{E^b}{c^2} = 0. \tag{19}$$

Solving the sytem of equations (19) for f_1 and $\mathbf{v}_1^2$ in terms of the binding energy E^b, we find that

$$f_1(E^b) = 1 - \frac{m_1^\infty s c^2}{E^b} + \frac{2m_1^{\infty 2} s(s+1)c^4 - 2E^b m_1^\infty (s+1)c^2 + (E^b)^2}{2E^b(E^b - c^2 m_1^\infty(s+1))}$$

and $\mathbf{v}_1^2(E^b)$

$$= -\frac{E^b\left(E^b - 2c^2 m_1^\infty\right)\left(4m_1^{\infty 2} s(s+1)c^4 - 2E^b m_1^\infty(2s+1)c^2 + (E^b)^2\right)}{4c^2 m_1^{\infty 2}\left(E^b - c^2 m_1^\infty(s+1)\right)^2}. \tag{20}$$

In the interesting special case when $m_2^\infty = s m_1^\infty$ and $s \to \infty$, we find that the velocity

$$\mathbf{v}_1^2 \to \frac{E^b(2c^2 m_1^\infty - E^b)}{c^2 (m_1^\infty)^2}. \tag{21}$$

We will use this result later.

Similarly, we can now obtain the instantaneous rest-masses

$$m_1(E^b) = m_1^\infty(1 - f_1\frac{E^b}{m_1^\infty c^2})$$

or

$$m_1(E^b) = m_1^\infty(1+s) - \frac{E^b}{c^2} - \frac{2m_1^{\infty 2} s(s+1)c^4 - 2m_1^\infty(s+1)E^b c^2 + (E^b)^2}{2c^2(E^b - c^2 m_1^\infty(s+1))} \tag{22}$$

and

$$m_2(E^b) = s m_1^\infty(1 - (1 - f_1)\frac{E^b}{s m_1^\infty c^2})$$

or

$$m_2(E^b) = \frac{2m_1^{\infty 2} s(s+1)c^4 - 2m_1^\infty(s+1)E^b c^2 + (E^b)^2}{2c^2(E^b - c^2 m_1^\infty(s+1))}. \tag{23}$$

We now calculate for what *critical value* E_c^b of the binding energy E^b the smaller mass $m_1(E_c^b) = 0$. We find that

$$E_c^b = c^2 m_1^\infty\left(s + 1 - \sqrt{s^2 - 1}\right).$$

For this value of the binding energy E^b, we find that

$$m_2(E_c^b) = m_1^\infty\sqrt{s^2 - 1}, \quad \mathbf{v}_1^2(E_c^b) = c^2, \quad \text{and} \quad \mathbf{v}_2^2(E^b) = \frac{\mathbf{v}_1^2(E^b)}{s^2}.$$

We also find that $f_1(E_c^b) = \frac{1}{1+s-\sqrt{s^2-1}}$.

It is interesting to graph the instantaneous rest-masses $m_i(E^b)$ for $i = 1, 2$, the velocity $|\mathbf{v}_1(E^b)|$ and the fraction $f_1(E^b)$ of the binding energy being consumed by the first mass, in terms of the total binding energy E^b being expended. In Figure 1, the velocity of light $c = 1$, the mass $m_1^\infty = 1$, $m_2^\infty = \sqrt{2}$, and the binding energy E^b satisfies the constraints $0 \le E^b \le \sqrt{2}$. At the critical value $E^b = \sqrt{2}$ the mass m_1^∞ has entirely consumed itself. Note that up to now, we have made no assumption regarding the nature of the force or forces which produce this binding energy. In the next section, we will assume that the binding energy is due to an inverse square law attractive force such as that due to Newton's law of gravitational attraction.

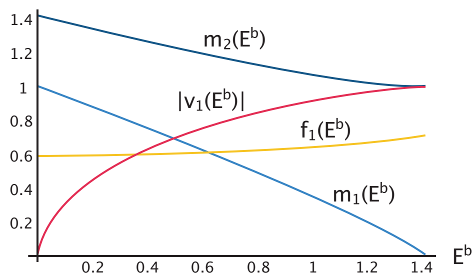

FIGURE 1. The masses $m_1(E^b)$ and $m_2(E^b)$, the velocity $|\mathbf{v}_1(E^b)|$ and $f_1(E^b)$ are plotted as functions of the binding energy E^b. Initially, $m_1(0) = 1$, and $m_2(0) = \sqrt{2}$.

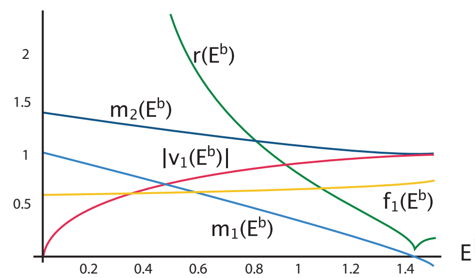

FIGURE 2. This figure is the same as figure 1, except that the numerical inverse solution $r(E^b)$ for the distance r between the two bodies, acted upon by the force of gravity, is shown as a function of the binding energy E^b. Note that the value of $r \to 0$ at exactly the moment the binding energy $E^b = \sqrt{2}$, and that $\lim_{E^b \to 0} r(E^b) = \infty$.

4. Binding energy due to Newton's gravitational force

In the case that the binding energy between the two bodies is totally due to Newton's gravitational attraction (12), we can write down the differential equation for the total binding energy $E^b(r)$ as a function of the distance r between the two bodies as measured in the rest-frame u. We get

$$\frac{dE^b}{dr} = -\frac{Gm_1(E^b(r))m_2(E^b(r))}{r^2} \tag{24}$$

where $m_1(E^b(r))$ and $m_2(E^b(r))$ are given in (22) and (23), respectively. Making these substitutions, we arrive at the rather complicated Riccati-like differential equation

$$\begin{gathered}
4G(m_1^\infty)^4 s(s+1)^2(2s+1)c^8 - 4G(m_1^\infty)^3(s+1)\left(5s^2+6s+1\right)E^b(r)c^6 \\
+2G(m_1^\infty)^2\left(11s^2+18s+7\right)(E^b(r))^2c^4 \\
-12G(m_1^\infty)(s+1)(E^b(r))^3c^2 + 3G(E^b(r))^4 \\
+\Big(-4(m_1^\infty)^2r^2(s+1)^2c^8 + 8(m_1^\infty)r^2(s+1)E^b(r)c^6 - 4r^2(E^b(r))^2c^4\Big)E^{b\prime}(r) \\
= 0.
\end{gathered}$$

We shall consider the solutions of various special cases of this differential equation.

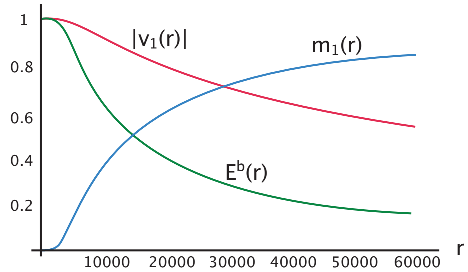

FIGURE 3. The mass $m_1(r)$, the binding energy $E^b(r)$, and the velocity $|\mathbf{v}_1|$ are shown for $0 \le r \le 60000$. This is the case of binding to a celestial body. To make this figure, we have assumed that $m_2^\infty = 10000 m_1^\infty$ where $m_1^\infty = 1$.

4.1. **Two body problem.** We first consider a numerical solution in the case that $m_1^\infty = 1$, $m_2 = s = \sqrt{2}$, and the constants $G = c = 1$. For this case, the graph of the solution is given in Figure 2. Note that we are actually plotting the *inverse function* $r(E^b)$ of the solution. This is permissible because, as can be seen in the figure, $r(E^b)$ is a *strictly decreasing* function in the physical range of interest for $0 < E^b \le \sqrt{2}$. Note also that $r(\sqrt{2}) = 0$, although the accuracy of the numerical solution does not clearly show this.

In the case that the body m_2^∞ is so massive that $m_2(r) = m_2^\infty$ for all values of $r \ge 0$, the differential equation (24) becomes

$$\frac{dE^b}{dr} = -m_2^\infty \frac{G m_1(r)}{r^2}, \tag{25}$$

which, together with the boundary condition that $E^b(\infty) = 0$, gives the particularly surprising solution

$$E^b(r) = E_1^b(r) = c^2(1 - e^{-\frac{Gm_2^\infty}{c^2 r}}) m_1^\infty,$$

or solving (17) for $m_1(r)$,

$$m_1(r) = e^{-\frac{Gm_2^\infty}{c^2 r}} m_1^\infty.$$

Using (21) and the expression for $E^b(r)$ above, we find the velocity

$$|\mathbf{v}_1(r)| = c\Big(1 - e^{\frac{-Gm_2^\infty}{c^2 r}}\Big).$$

See Figure 3. The differential equation (25) and its solution, were first derived in [43], and a discussion of how it is related to the total energy found by Einstein can be found therein.

Another interesting two body case is when the masses $m_1^\infty = m_2^\infty$. In this case the differential equation for the binding energy becomes

$$\frac{dE^b(r)}{dr} = 2\frac{dE_1^b}{dr} = -\frac{Gm_1^2(r)}{r^2} = -\frac{G(m_1^\infty - \frac{E_1^b(r)}{c^2})^2}{r^2}, \tag{26}$$

which has the simple solution

$$E^b(r) = \frac{2c^2 G (m_1^\infty)^2}{Gm_1^\infty + 2c^2 r}.$$

We also easily find

$$m_1(r) = m_1^\infty - \frac{E_1^b(r)}{c^2} = \frac{2c^2 m_1^\infty r}{Gm_1^\infty + 2c^2 r},$$

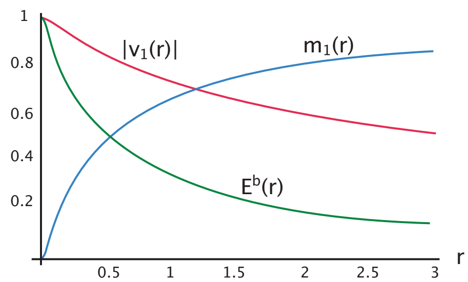

FIGURE 4. The mass $m_1(r) = m_2(r)$, the binding energy $E_1^b(r)$, and the velocity $|\mathbf{v}_1(r)|$ are shown for $0 \leq r \leq 3$.

and using (20), the velocity

$$|\mathbf{v}_1(r)| = c\frac{\sqrt{Gm_1^\infty(Gm_1^\infty + 4c^2r)}}{Gm_1^\infty + 2c^2r}.$$

See Figure 4. The terminal velocities of the equal bodies $m_1(r)$ and $m_2(r)$, when they self-annihilate, are equal to the speed of light c.

4.2. **Three body problem.** As might be expected, the general three body problem is harder than the two body problem. However, we can solve the three body problem in the special case that $m_1^\infty = m_2^\infty$ and the third body m_3^∞ lies on a line between m_1^∞ and m_2^∞. We can immediately write down the rigorous differential equation for the binding energy E^b for this system. We find that

$$\frac{dE^b(r)}{dr} = -2\frac{Gm_3^\infty m_1^\infty}{r^2}\Big(1 - \frac{E^b(r)}{2m_1^\infty c^2}\Big) - \frac{G(m_1^\infty)^2}{4r^2}\Big(1 - \frac{E^b(r)}{2m_1^\infty c^2}\Big)^2, \tag{27}$$

which has the closed form solution

$$E^b(r) = \frac{2\,c^2\left(1 - e^{\frac{Gm_3^\infty}{c^2 r}}\right)m_1^\infty\,(m_1^\infty + 8m_3^\infty)}{m_1^\infty - e^{\frac{Gm_3^\infty}{c^2 r}}\,(m_1^\infty + 8m_3^\infty)}. \tag{28}$$

Note that in writing down this differential equation, we have used the fact that when the masses $m_1(r)$ and $m_2(r)$ are at a distance of r from the mass m_3^∞ in the center, we have the relationship that

$$m_1(r) = m_1^\infty\Big(1 - \frac{E^b(r)}{2m_1^\infty c^2}\Big) = m_2(r).$$

From (28) and this last relationship, it follows that

$$m_1(r) = \frac{8m_1^\infty m_3^\infty}{-m_1^\infty + e^{\frac{Gm_3^\infty}{c^2 r}}\,(m_1^\infty + 8m_3^\infty)}.$$

The graph given in Figure 5 plots the mass of $m_1(r)$ on the y-axis for values of r on the x-axis for $r \geq 0$. We have taken $m_1^\infty = 1 = m_2^\infty, m_3^\infty = 3, c = 1, G = 1$ to produce this graph. The critical value when $m_1(r) = 0$ occurs when $r = 0$.

It is not surprising that Figure 3 and Figure 5 are very similar. Indeed, for $m_3^\infty >> m_1^\infty = m_2^\infty$, the formulas for the masses $m_1(r)$ become the same. Both Figure 3 and Figure 5 strongly suggest that black holes do not exist. Whenever a less massive object approaches a very massive object, depending upon initial conditions, it will necessarily self-annihilate or coalesce. There cannot be any critical mass which would define the *Schwarzschild radius* of a black hole.

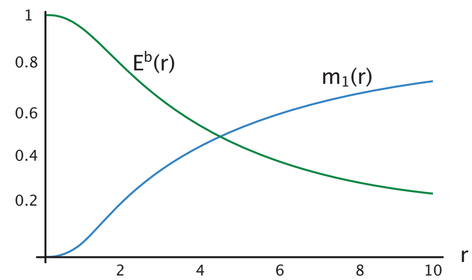

FIGURE 5. The two masses $m_1(r)$ and the binding energy $E^b(r)$ are shown for $0 \leq r \leq 10$. Both masses $m_1^\infty = m_2^\infty = 1$ self annihilate as they approach the mass $m_2^\infty = 3$ at $r = 0$.

5. Discussion

A major problem of general relativity is that it does not easily lend itself to quantization, although Einstein, himself, apparently did not believe in quantum mechanics [12]. We have seen that, theoretically, when a mass falls from infinity into a larger mass it will self-annihilate at $r = 0$. However, quantum mechanics implies that the object's dimensions effectively become that of space itself at $r = 0$. The restrictions of quantum mechanics imply, therefore, that r can never reach the value $r = 0$ for macroscopic objects. In the case of elementary particles, where additional forces other than gravity are known to be at work, self-annihiliation *does* occur. We have already seen that in our approach singularities, even those arising from the inverse square dependency of Newton's Law, disappear.

Indeed, taking into account how unit lengths quantum mechanically stretch in a gravitational field, the second author obtained the *precession of the perihelion of Mercury* as well as the *deflection of light* passing near a celestial body [43]. Typically, these have been considered to be the best proofs of the validity of Einstein's general theory of relativity. In addition, our approach nicely lends itself to the quantization of gravitation [42], and of any other field that the object in question can interact with, since, as we have seen in Section 3, the concept of binding energy is in no way restricted to gravitational forces [39], [40], [41]. In fact, it yields in a straightforward way the de Broglie relationship [45].

A consequence of our theory is that black holes of macroscopic objects *solely due to the force of gravity* do not exist. Rather, when a sufficient amount of mass coalesces in space, the object becomes either invisible or nearly invisible due to the extreme red-shift near such a body. We thus predict that very dark objects, but no black holes, should be found in the center of many galaxies. On the other hand, if a sufficient amount of mass coalesces causing a total collapse to values of r so small that other elementary forces become predominant, then it becomes plausible that there will be a partial or even a total annihiliation of the macroscopic body with a corresponding large burst of energy. This may explain the presence of the recently discovered "biggest expanse of nothing", a billion light years wide, which is the space that would normally be occupied by thousands of galaxies. "No stars, no galaxies, no anything" [30].

Although our theory produces results that are practically the same as those of the General Theory of Relativity, they are only the same up to a third order Taylor expansion. Ultimately, the value of any theory rests not upon the conviction or authority of its authors, but on the fruits of its predictions and its ability to encompass and explain experimental results.

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