

***THE GENERAL EQUATION OF MOTION VIA THE SPECIAL  
THEORY OF RELATIVITY AND QUANTUM MECHANICS  
PART II: CHECK AGAINST THE BASIC PREDICTIONS OF  
THE GENERAL THEORY OF RELATIVITY***

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## ABSTRACT

In Part I of this work, we derived a *general equation of motion*, based only on the *special theory of relativity* and *energy conservation*. This equation, turned out to be that of Newton, in the case the motion is driven by a *weak gravitational field*, with a *velocity small as compared to the velocity of light*. Thus in Part I we found

$$\frac{e^{-\alpha_0(r_0)}}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} = \text{Constant} ; \quad \alpha_0(r_0) = \frac{GM_0}{r_0 c_0^2} ;$$

or, by *differenciation*,

$$-\frac{GM_0}{r_0^2} \left( 1 - \frac{v_0^2}{c_0^2} \right) = v_0 \frac{dv_0}{dr_0} ;$$

(written by the author, in the local frame of reference)

here  $r_0$  is the distance of the object to the center of celestial object of mass  $M_0$ ,  $v_0$  its velocity, as referred to the local observer;  $G$  is the universal constant of gravitation, and  $c_0$  the velocity of light in empty space.

The above equation is written for the *local observer*; we should as well be able to write it, as seen by the *distant observer*. Thus, as we have discussed, the *rest mass* of an object in a gravitational field (*in fact in any field the object in hand enters into interaction*), is decreased as much as its *binding energy* in the field; a *mass deficiency* conversely, via *quantum mechanics*, yields the *widening of the period of time* the object displays, the *stretching of its size*, as well as the *weakening of its internal energy*. Henceforth we are not in the need of the “*principle of equivalence*” assumed by the *general theory of relativity*, in order to predict the occurrences dealt with this theory.

Our approach then, as viewed by the *distant observer*, yields

$$-\frac{GM_0}{r^2} e^{-\alpha_0} \left( 1 - 2 \frac{v^2}{c_0^2} e^{2\alpha_0} \right) = v \frac{dv}{dr} ; \quad \alpha_0 = \frac{GM_0}{r_0 c_0^2} ;$$

here  $r$  is the distance of the object to the center of celestial object of mass  $M_0$ , and  $v$  its velocity, as referred to the distant observer.

The frame drawn by the above equation allows us to derive the essential findings of the general theory of relativity, i.e. the bending of light through its passage nearby a celestial body, and the precession of the perihelion of the planets. Thus light is deflected exactly twice of what is classically predicted, whereas we predict for Mercury, a precession of the perihelion about 1.3% less than what Einstein predicted; the difference in question is experimentally indiscernible in the case of Mercury, but it should become more important, in the case of a celestial body moving in a stronger field, also on an orbit with higher eccentricity than that of Mercury.

Following our approach we further undertake the behavior of an object thrown with a very high speed from a celestial body; thus the speed decreases exponentially to reach an asymptotic value.

## 1. INTRODUCTION

In the first part of this work we derived a general equation of motion, based only on the *special theory of relativity* and *energy conservation*. This equation, turned out to be that of Newton, in the case the motion is driven by a weak gravitational field, with a velocity small as compared to the velocity of light.

Thus we found

$$\frac{e^{-GM_0/r_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Constant} ; \quad (1-a)$$

or, by *differenciation*,

$$-\frac{GM_0}{r_0^2} \left(1 - \frac{v_0^2}{c_0^2}\right) = v_0 \frac{dv_0}{dr_0} ; \quad (1-b)$$

(written by the author, in the local frame of reference)

here  $r_0$  is the distance of the object to the center of celestial object of mass  $M_0$ ,  $v_0$  its velocity, as referred to the local observer;  $G$  is the universal constant of gravitation, and  $c_0$  the velocity of light in empty space.

Recall that Eqs. (1-a) and (1-b) is written for the *local observer*. We should be able to write it, as assessed by the *distant observer*; indeed our metric, is (*just like the one used by the general theory of relativity*) altered by the gravitational field (*in fact, by any field the “measurement unit” in hand, interacts with*); yet in our approach, this occurs via *quantum mechanics*.

This is detailed in Part I (cf. Theorems 1 and 2); thus, as we have discussed, the *rest mass* of an object in a gravitational field, is decreased as much as its binding energy in the field; a *mass deficiency* conversely, via *quantum mechanics*, yields a *stretching* of its size, as well as the *weakening* of its internal energy.

Henceforth we do not need the “*principle of equivalence*” assumed by the *general theory of relativity*, in order to predict the occurrences dealt with this theory.<sup>1</sup> We predict them through our “*general equation of motion*”, essentially based on the special theory of relativity.

Thus below, we shall first elaborate on Eq.(1-a), to see how a motion described by this equation, is pictured by a *distant observer* (Section 2). Next, we check this equation against the *major predictions of the general theory of relativity*, i.e. basically the precession of the perihelion of a planet, and the deflection of light nearby a star (Sections 3-5). Then, we undertake the behavior of an object thrown with a very high speed from a celestial body (Section 6), followed by a conclusion (Section 7).

## 2. ELABORATION OF THE GENERAL EQUATION OF GRAVITATIONAL MOTION

In order to be able to carry a comparison of Eq.(1-b), with the classical Newtonian equation (*written out of this equation in the case where  $v_0/c_0$ , or  $\alpha_0$  is negligible as compared to unity*), we will now elaborate on  $v_0$ , and accordingly on  $dv_0/dr_0$ .

To ease the phrasing of our analysis, let us (*without any loss of generality*), go back to our specific problem, i.e. the motion of Mercury around the sun.

*Classically*,  $r_0$  is the distance of the planet to the sun at the location in question, no matter whether it is assessed by the *distant observer* or by the *local observer* (*affected by the gravitational field*). According to the present approach,  $r_0$  is still the distance of the planet to the sun at the given location; yet this distance appears to be different when assessed by the distant observer; it is not  $r_0$  but  $r$ , a *greater quantity* than  $r_0$ , as shall be detailed soon.

Likewise *classically*,  $v_0(r_0)$  is the velocity of Mercury on the orbit, regardless whether it is measured by the *distant observer*, or the *local observer*, affected by the gravitational field. According to our approach (*as we will precise below*),  $v_0(r_0)$  is still the velocity of the planet as measured by the local observer (*affected by the gravitational field*); yet the velocity of the planet according to the distant observer is not the same; we shall call it  $v(r_0)$ , a *smaller quantity* than  $v_0(r_0)$ .

To clarify how we should relate  $v(r)$  and  $v_0(r_0)$ , we would like to recall a previous discussion<sup>2</sup> we carried about the “*slowing down of the velocity of light in the vicinity of a celestial body*” (Appendix B of the cited reference), as implied by Theorem 2 of Part I.

Our approach goes as follows (cf. Figure 1).

According to our Theorem 2, a *stick meter* of length  $\mathfrak{R}_0$  in empty space (*contrary to what the general theory of relativity predicts*), stretches, nearby a celestial body, to become  $\mathfrak{R}$ . Thus, following Theorem 1, and just like Eq.(11) of Part I, we can write<sup>\*</sup>

$$\mathfrak{R} = \mathfrak{R}_0 e^{\alpha_0(r_0)}. \quad (2)$$

where,  $\alpha_0(r_0)$  or in short  $\alpha_0$  is  $\mathbf{GM}_0 / r_0 c_0^2$  [cf. Eq.(10) of Part I].

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<sup>\*</sup> Note that the general theory of relativity, instead, predicts

$$\mathfrak{R} = \mathfrak{R}_0 \sqrt{1 - 2\alpha_0} \cong \mathfrak{R}_0 (1 - \alpha_0), \quad (i)$$

the approximation, being valid for a weak gravitational field. To our knowledge, this is however, not checked against any experimental result.

So, as light travels, from  $-\infty$  to  $+\infty$ , passing nearby the celestial body, it crosses a “unit path”  $\mathcal{R}(r)$  (picked around the location  $r$ ), instead of  $\mathcal{R}_0(r)$ , the *original length*, it would otherwise cross (*were there, no gravitational field around*). Thus, light in the presence of the gravitational field, as viewed by a *distant observer*, always traveling with the *usual constant speed*  $c_0$ , would yet spend a *longer time*, through the itinerary in question. This makes that, in the presence of the celestial body, *as viewed by the distant observer, were he unaware of the stretching of space*, light slows down as much.

In other words, the “*stretching of space*” and the “*slowing down of light*”, in the presence of a gravitational field, are *equivalent occurrences*.

The local observer though, is not capable to detect any difference, since *unit lengths* and *unit periods* (based on our Theorems 1 and 2, stated in Part I) dilate equally.

Henceforth, in our approach, the *slowing down process of light nearby a celestial body*, resembles to that *seemingly* drawn by *light*, within the frame of the special theory of relativity, *for example* traveling in between *two mirrors*, in a *light clock* of length  $L_0$ , brought to a *uniform translational motion* of velocity  $w_{tr}$ , in (*for simplicity*) say, a *direction perpendicular to the travel direction of light, in the clock*, as viewed by the *fixed outside observer*; thus in this example,  $L_0$  is not altered due to the motion. Let then  $T_0$  be the *back and forth travel period of time* of light in the *light clock*, at rest.  $T_0$ , *due to the motion*, as assessed by the outside fixed observer, becomes  $T = \gamma T_0$ , where as usual  $\gamma = 1/\sqrt{1 - w_{tr}^2/c_0^2}$ . Obviously  $c_0$  is an *invariant*; nonetheless we can define a *velocity*  $c$  as  $2L_0/T$ , i.e.  $c_0/\gamma$ , which makes that the *stretching of the itinerary of light*, and the resulting *time dilation* due to the translational motion in consideration, can be considered for a *light slowing down*, if one wanted to keep the distance crossed by light, unaltered. (*The direction of the translational motion of the light clock, does not affect this result.*)

Just likewise, because (based on our Theorems 1 and 2, stated in Part I), *unit lengths loosen* nearby a celestial body; according to a *distant observer*, light grazing a celestial body, crosses a *longer distance* compared to what it would do, if there were no gravitation. This becomes clearer if we consider a *light clock* in a gravitational field; because the *box* in which *light* goes back and forth, shall stretch in the gravitational field; according to the *distant observer*, the *ticks* of the light clock in hand, shall appear weakened, *as if light (in the box) has slowed down as much*.<sup>†</sup>

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<sup>†</sup> It should be emphasized that, the *internal motion*, say of atomistic or molecular objects is drawn by only the *electric charges*. This is to say, it does not depend on the *change of the mass* of the object, if this somehow, ever happened. The *Bohr Atom Model*, may help to picture this fact, without any loss of generality. The *revolution speed*  $v$  of the electron around the proton is (*in CGS unit system*) given by  $v = 2\pi e^2/nh$ , where  $e$  is the charge intensity of either the electron or the proton,  $h$  the *Planck Constant*, and  $n$  the *principal quantum number*. Thus, this speed does not depend on the *mass of the electron*, nor obviously on any *perturbation* this may undergo. It is interesting to note that for instance in the case the electron is replaced by the *muon*, bearing the same charge, but a mass about 207 times greater than that of the electron, the revolution speed of it around the nucleus, shall still remain the same, though the muon shall be bound to the proton at a distance as much closer. Note further that the *two ingredients* of Bohr Atom Model, i.e.  $e^2/r^2 = mv^2/r$  and  $2\pi nvr = nh$  (*where  $r$  is the radius of the electron's orbit and  $m$  the electron mass*), are well compatible with the special theory of relativity (*the present theory's essential basis*), i.e. relationships one shall derive out of these two equations, about the two unknowns  $v$  and  $r$ , remain *Lorentz invariant*.

We would like to stress that in our approach, the value of  $c_0$ , the speed of light in empty space, as measured by the *distant observer*, on the contrary to the actual wisdom,<sup>1</sup> developed within the frame of the *general theory of relativity*, is not altered.

Though, because of the *stretching* of the itinerary in question, according to *the distant observer*, light shall be viewed *as if*, it slows down, in the presence of a gravitational field.

Let than  $c(r)$ , the velocity of light at  $r$ , in the gravitational field, as viewed by the *distant observer*. According to the foregoing reasoning [essentially based on Eq.(2)],  $c(r)$  is given by

$$c(r) = c_0 e^{-\alpha_0(r_0)} . \quad (3)$$

At this stage let us go back to our *light clock*, in which we had considered light traveling in between two mirrors, brought to a *uniform translational motion* of velocity  $w_{tr}$ , as viewed by a *fixed outside observer*.

If in fact, this were not a *light clock*, but any clock, say a box *at rest*, in which an “*ordinary clock particle*”, which we may call “*clock pendulum*”, moves back and forth with the velocity  $u_{0inside}$ ; the *clock*, based on the “*principle of relativity*”, would still retard as much as  $\gamma = 1/\sqrt{1-w_{tr}^2/c_0^2}$ , when brought to a uniform translational motion (of velocity  $w_{tr}$ ), through which  $u_{0inside}$  becomes  $u_{inside}$ , as assessed by the *fixed outside observer*.

Thus the velocity  $u_{inside}$  of the “*clock pendulum*” motion, in the *moving frame*, as assessed by the fixed outside observer shall be given by

$$u_{inside} = \frac{u_{0inside}}{\gamma} . \quad (4)$$

Likewise, if now we consider the “*box*” in question (*bearing an “ordinary clock particle” going back and forth*), in a gravitational field, because the *box* shall (*due to the gravitation*) stretch, the *ticks* of the clock in hand, shall according to the *distant observer*, appear weakened, *as if* the particle inside the box has slowed down as much.

Accordingly our Eq.(3) in a gravitational field, shall not be written for only the *speed of light*, but can well be generalized [just like what we did at the level of Eq.(3)] for any *velocity*, the *clock pendulum* would display.

Let us especially consider again, the motion of Mercury around the sun, and precise the following definitions.

- $r_0$  or  $r_0(t_0)$  : the distance of the planet to the sun, at time  $t_0$ , as assessed by the *local observer*
- $r$  or  $r(t)$  : the distance of the planet to the sun, as assessed by the *distant observer*, at time  $t$  (*still measured in the same frame*)
- $v_0(r_0)$ , or in short  $v_0$  : the velocity of Mercury, at the location  $r_0$  on the orbit, as assessed by the *local observer*

$v(r)$ , or in short  $v$  : the velocity of Mercury, at the location  $r$  on the orbit, as assessed by the *distant observer*

Note that just like the *local observer* measures the *speed of light* as  $c_0$  (*the very speed of light in empty space*); similarly,  $v_0$  measured by the *local observer*, is the velocity, the *distant observer* would measure *if there were no gravitation*.

Thence, based on the foregoing discussion, and particularly Eq.(3), we have

$$v(r) = v_0(r_0) e^{-\alpha_0(r_0)}. \quad (5)$$

Recall that, the velocity appearing in Eq.(1) [the same, in Eq.(13) of Part I], is well  $v_0(r)$ ; i.e. following the above equation, one can write

$$v_0(r) = v(r) e^{\alpha_0}. \quad (6)$$

We can now transform Eq.(1) to predict, what the distant observer shall assess.

Thus, following Eq.(2), we have

$$dv_0(r_0) = dv(r) e^{\alpha_0} + v(r) d\alpha_0(r_0) e^{\alpha_0}. \quad (7)$$

On the other hand one can, following the definition of  $\alpha_0$  [cf. Eq.(10) of Part I], write

$$\frac{d\alpha_0}{dr_0} = -\frac{GM_0}{c_0^2 r_0^2}; \quad (8)$$

Note that here  $dr_0$  is the infinitely small distance (around  $r_0$ ), as measured by the *local observer*, and  $dr$  is the *same element* as measured by the *distant observer*. Yet the distant observer measures the quantity  $dr_0$ , as  $dr$ .

According to Eq.(2), the relationship between these two *differential elements*, becomes

$$dr = dr_0 e^{\alpha_0(r_0)}. \quad (9)$$

Eqs. (5) - (8) enable us to transform Eq.(1), the *general equation of gravitational motion*, written for the local observer, i.e.

$$d\alpha_0 \left( 1 - \frac{v_0^2}{c_0^2} \right) = \frac{v_0}{c_0^2} dv_0. \quad (10)$$

Eq.(9), on the other hand; regarding the *distance of Mercury to the sun's center*, as assessed by the *distant observer*, yields

$$r = r_0' e^{\alpha_0'(r_0')} \Big|_0^{r_0} - \int_0^{R_0} r_0' e^{\alpha_0'(r_0')} d\alpha_0' - \int_{R_0}^{r_0} r_0' e^{\alpha_0'(r_0')} d\alpha_0' , \quad (11)$$

where  $R_0$  is the *sun's proper radius*, and  $\alpha_0'$  is a simplified representation for  $\alpha_0'(r_0')$ .

One can show that for the case of Mercury, the above equation reduces to<sup>‡</sup>

$$r \cong r_0 e^{\alpha_0} . \quad (12)$$

Accordingly, Eq.(10) becomes

$$-\frac{GM_0}{r^2} e^{-\alpha_0} \left( 1 - 2 \frac{v^2}{c_0^2} e^{2\alpha_0} \right) = v \frac{dv}{dr} . \quad (13)$$

(equation of motion written by the author for Mercury,  
in fact for any object, as assessed by the distant observer)

Note that this equation reduces well to the *classical Newtonian equation*, if  $v$  can be neglected as compared to the *velocity of light in empty space*, and if  $\alpha_0$  is small.

Eq.(13) on the other hand [via Eq.(9)], *fortunately* reduces to the *differential form* of Eq.(3) [which is the *basic ingredient* in transforming our *original equation*, i.e. Eq.(1-b) (written for the *local observer*), into Eq.(13), written for the *distant observer*], when  $v_0$  is set to  $c_0$ , and  $v$  is set to  $c$ , confirming the *internal rigor* of our approach.

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<sup>‡</sup> We can show that within the sun  $\alpha_0'(r_0')$ , or in short  $\alpha_0'$ , becomes<sup>2</sup>

$$\alpha_0'(r_0') = \alpha_0'(R_0) + \frac{GM_0}{2c_0^2 R_0^3} (R_0^2 - r_0'^2) . \quad (i)$$

Thus the *first integral* at the RHS of Eq.(11), is in the order of  $R_0$ , the *radius of the sun*, and is anyway *negligible* as compared to  $r_0$ , the *radius of the orbit of Mercury*. The *second integral* at the RHS of Eq.(11) can, along Eq.(9), be written as

$$\int_{R_0}^{r_0} r_0' e^{\alpha_0'(r_0')} d\alpha_0' \approx \frac{GM_0}{c_0^2} \left[ \ln \alpha_0'(r_0') \right] \Big|_{R_0}^{r_0} ; \quad (ii)$$

the outcome (*though a positive contribution*), is in the order of  $r_0 \alpha_0(r_0)$  (*written at the location of Mercury*), and the effect of this quantity next to  $r_0$  is practically *none*. Therefore the result of Eq.(11) is well, Eq.(12) (c.q.f.d.).



Following Eqs. (18) and (19) of Part I (*i.e. the assumptions of small  $\alpha_0$  and circular orbit*), the above equation, reduces to

$$-\frac{GM_0}{r^2} \left( 1 - 3 \frac{v^2}{c_0^2} e^{2\alpha_0} \right) = v \frac{dv}{dr} , \quad (14)$$

(*approximate equation of motion written by the author for Mercury as assessed by the distant observer*)

which (*though not exactly the same*) appears, for the case of Mercury, in a *desirable congeniality* with the *equation of motion predicted by the general theory of relativity*.<sup>1,3</sup>

A more detailed study and a numerical application about the subject shall be presented below.

Note that Eq.(13) yields

$$\frac{dv}{dr} < 0, \text{ if } v_0 < \frac{c_0}{\sqrt{2}} , \quad (15)$$

and

$$\frac{dv}{dr} > 0, \text{ if } v_0 > \frac{c_0}{\sqrt{2}} . \quad (16)$$

Eq.(15) points to the *usual occurrence*; indeed, as an object moves toward a gravitational source, its speed, through the interaction, *as assessed by the distant observer*, shall increase. (*Note that here,  $dv$  and  $dr$  display opposite signs.*)

Eq.(16) though unusual, discloses the fact that, if the object of concern, approaches a gravitational source with a speed greater than  $c_0/\sqrt{2}$ , then its velocity, through the interaction, still *as assessed by the distant observer*, shall decrease. An object passing by, with a speed just equal to  $c_0/\sqrt{2}$  *as assessed by the distant observer*, shall keep on going with the same speed, no matter how intense is the gravitational field (*though of course there will still be bending, toward the gravitational source*).

### 3. HOW IS THE PATH DRAWN?

In our approach, the bending of the trajectory of an object (*in motion, in a direction different than that of the gravitational force*), nearby a celestial body, as assessed by the *distant observer*, is clearly due to *two distinct processes*.

The first one, according to our *postulate* about *energy conservation* (stated in Part I of this work), is due to the *variation of the rest mass* of the object in consideration (*though at an infinitesimal rate, in most cases we observe*). Thus, while the object piles up, as *additional rest mass*, some of its *kinetic energy* through a *slowing down course*, or loses some of its *rest mass*, increasing its *kinetic energy* in a *run away course*, it should, due to the *conservation of the linear momentum*, receive kicks, which in fact, must be the cause of the acceleration (*either the slowing down or the run away courses*), in question.

This tacit process is in fact, the same as that of the *Classical Newton's pull*, since this was a main ingredient to our approach, except that we considered the “energy conservation” (not in the classical, but) in the broader sense.

The *second process* responsible of the bending in question (still, as assessed by the distant observer), is so to say the *quantum mechanical alteration of its speed by the gravitational field*; its *local velocity*, whatever it is, appears slower to the *distant observer* [cf. Eq.(3), or in general Eq.(5)]. In fact, this process alone, though through a *different philosophy* is considered by the *general theory of relativity* (where moreover, as shall be elaborated below, the “strength” of the slowing down of light, for instance, in a gravitational field is, twice as that we predict, and this, were the field weak). The resulting bending then, is calculated via the application of the Fermat Principle.<sup>1,2</sup>

The two processes as we disclose, are linked to each other, as much as *conservation of energy* and *quantum mechanics* are.

Below we are going to consider two well known cases, i.e. *the deflection of light* nearby the sun, and the *precession of the orbit* of Mercury.

Based on the foregoing discussion we can handle these two cases, by seeking a solution of Eq.(14) in relation to the classical *Newton Equation*, i.e.

$$\frac{GM_0}{r^2} \frac{r}{v} = \frac{dv}{dt} \quad , \quad (17)$$

or in scalar form,

$$-\frac{GM_0}{r^2} = v \frac{dv}{dr} \quad . \quad (18)$$

The *extra bending*  $\mathfrak{G}_{\text{extra}}$  implied by our approach as compared to the *classical Newtonian bending*, can then be easily determined along Fermat principle, through the following difference

$$\mathfrak{G}_{\text{extra}} = \int_{\text{path}} -\cot \text{an} \theta \frac{dv}{v} \Big|_{\text{extra}} = \int_{\text{path}} -\cot \text{an} \theta \frac{dv}{v} \Big|_{\text{Author}} - \int_{\text{path}} -\cot \text{an} \theta \frac{dv}{v} \Big|_{\text{Newton}} \quad ; \quad (19)$$

here  $\theta$  is the angle the *path* makes with the *gravitational field line*.

#### 4. DEFLECTION OF LIGHT NEARBY THE SUN, THROUGH THE PRESENT APPROACH

Based on Eq.(19), we first undertake the problem of light (cf. Figure 1) through its passage nearby the *sun*, without any loss of generality.

Note that classically the speed of light  $c_0$ , is not altered nearby a gravitational field, i.e. *classically*  $dc/c$  is zero.

Thus, via Eqs. (3) and (19), we obtain

$$\mathfrak{G}_{\text{extra}} = -\int_{-\infty}^{\infty} \frac{dc(z)}{c(z)} \frac{\sin \theta}{\cos \theta} = -\int_{-\infty}^{\infty} d\alpha_0(z) \frac{R_0}{z} = \int_{-\infty}^{\infty} \frac{GM_0}{c_0^2} \frac{R_0 dz}{(R_0^2 + z^2)^{3/2}} = 2 \frac{GM_0}{c_0^2} = 0.875'' . \quad (20)$$

Therefore  $\mathfrak{G}_{\text{extra}}$  turns out to be as much as the *classical bending*; this makes that according to our approach light, through its passage nearby the sun is deflected twice as much as the deflection predicted by the Newtonian approach, i.e. 1.75'' altogether (c.q.f.d.).

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Recall that the general theory of relativity considers a *contraction of lengths, instead of a stretching (contrary to what we proposed herein)*, and a *conjoint dilation of time*, in a gravitational field; accordingly, for the case of the light deflection nearby the sun, Einstein (*though we still object, as discussed right below*), wrote<sup>1</sup>

$$c(z) = c_0 \frac{1 - \alpha_0(z_0)}{1 + \alpha_0(z_0)} \cong c_0 [1 - 2\alpha_0(z_0)] \quad (21)$$

(written by Einstein for the light slowing down, nearby the sun),

versus

$$c(z) = c_0 e^{-\alpha_0(z_0)} \cong c_0 [1 - \alpha_0(z_0)] . \quad (22)$$

(written by the author for the light slowing down, nearby the sun)

Thus following Einstein, the resulting deflection angle  $\mathfrak{G}$  becomes

$$\mathfrak{G} = -\int_{-\infty}^{\infty} \frac{dc(z)}{c(z)} \frac{R_0}{z} = -\int_{-\infty}^{\infty} 2d\alpha_0(z) \frac{R_0}{z} = \int_{-\infty}^{\infty} \frac{2GM_0}{c_0^2} \frac{R_0 dz}{(R_0^2 + z^2)^{3/2}} = 4\alpha_0 . \quad (23)$$

(written by Einstein)

Note that even if the structure of Eq.(21) is considered to be consistent with the idea of contraction of lengths, next to the time dilation; this equation still seems to be *erroneous*, and this, already based on the general theory of relativity. Indeed, as mentioned above, according to this theory, a stick of length  $R_0$  in the gravitational field of the sun, becomes  $R_0[1 - \alpha_0(z_0)]$ , only so long as it is held parallel to the radius of the sun; a stick that is perpendicular to the radius, behaves normally, as observed by the distant observer.<sup>1, 3</sup> This makes that, a stick of length  $R_0$  placed at the location  $z$ , along the path of light nearby the sun (cf. Figure 1), is in fact (according to the general theory of relativity), shortened by less than a factor of  $[1 - \alpha_0(z_0)]$ ; this factor, more precisely is  $[1 - \alpha_0(z_0)] (|z_0|/r_0)$ ; thence Eq.(21) is clearly inconsistent with the frame of the general theory of relativity. The result, were the necessary correction is taken into account, surprisingly comes out to be half of what the general theory of relativity claimed, thus in fact, the classical result (still via the general theory of relativity).

Our approach furthermore, regarding the *light deflection problem* is consistent with the results Pond and Rebka measured, in regards to *light frequency change* in a gravitational field<sup>4</sup> (*since this occurrence is essentially due to the energy conservation thus the increase of the energy of the falling photon in the gravitational field, and “energy conservation” is the main ingredient to our approach*). The general theory of relativity does not predict any such change.

## 5. A DETAILED STUDY ABOUT THE PRECESSION OF THE PERIHELION OF MERCURY

Here we shall apply Eq.(19) to predict how much Mercury (*according to our approach*), is deflected from the Newtonian trajectory, as observed by a *distant observer*.

Thus, according to our approach, the *relative differential change in the speed of the planet*, becomes

$$\left. \frac{dv}{v} \right|_{\text{Author}} = \frac{c_0^2}{v^2} e^{-\alpha_0} \left( 1 - 2 \frac{v^2}{c_0^2} e^{2\alpha_0} \right) d\alpha, \quad (24)$$

versus the corresponding, *classical Newton formulation*, i.e.

$$\left. \frac{dv}{v} \right|_{\text{Newton}} = \frac{c_0^2}{v^2} d\alpha. \quad (25)$$

The precession  $\vartheta_p$  of the perihelion of Mercury, through each revolution is then

$$\vartheta_p = \oint -\cot \text{an}\theta \left. \frac{dv}{v} \right|_{\text{Author}} - \oint -\cot \text{an}\theta \left. \frac{dv}{v} \right|_{\text{Newton}}; \quad (26)$$

here  $\theta$  is again, the angle the *path* makes with the *gravitational field line*.

Thus based on Eqs. (24), (25), the *Taylor expansion of the exponential term* in the case of *weak gravitation*, and our original equation of motion, Eq.(13) of Part I, as well as the equations derived from this, i.e. Eqs. (18) and (20) (of Part I), in the case of still *weak gravitation*, also a *slow cruise velocity*,  $d\vartheta_p$  becomes

$$d\vartheta_p = \frac{c_0^2}{v^2} \cot \text{an}\theta \left( \alpha_0 + 2 \frac{v^2}{c_0^2} \right) d\alpha = \cot \text{an}\theta \left( 2d\alpha + \frac{r}{a} d\alpha \right); \quad (27)$$

here  $a$  is the *semi-major axis* of the classical orbit; right below, we shall further call  $b$ , the corresponding *semi-minor axis*.

Thence, the divergence  $\vartheta_p$  between the *actual orbit* and the classical (*closed*) elliptic orbit, through each revolution of the planet around the sun, is given by

$$\vartheta_p = \oint_{\text{orbit}} 2 \cot \text{an}\theta d\alpha + \oint_{\text{orbit}} \frac{r}{a} \cot \text{an}\theta d\alpha. \quad (28)$$

The two integrals appearing in here, can be calculated to yield<sup>§</sup>

$$\vartheta_{P1} = \oint_{\text{orbit}} 2 \cot \text{an} \theta \, d\alpha = 2 \left( 2\pi \frac{GM_0 a}{b^2 c_0^2} \right), \quad (29)$$

$$\vartheta_{P2} = \oint_{\text{orbit}} \frac{r}{a} \cot \text{an} \theta \, d\alpha = 2\pi \frac{GM_0}{ac_0^2}. \quad (30)$$

This makes that

$$\vartheta_p = 2\pi \frac{GM_0}{c_0^2} \left( \frac{2a}{b^2} + \frac{1}{a} \right). \quad (31)$$

*(the rotation angle of the major axis of Mercury's orbit,  
through each revolution, derived by the author)*

The general theory of relativity, instead, predicts<sup>1</sup>

$$\vartheta = 6\pi \frac{GM_0 a}{b^2 c_0^2}, \quad (32)$$

*(the rotation angle of the major axis of Mercury's orbit,  
through each revolution, derived by Einstein)*

which happens to be just a little more than that predicted by the theory presented herein.

The difference of around 1.3% in between the two predictions appears to be experimentally indiscernible, in the case of Mercury, given that the uncertainties embodied by the latest radar measurements amount well to a couple of percent.<sup>5</sup> The difference in question should though become more important, in a stronger gravitational field, together with a greater eccentricity.

## 6. BEHAVIOR OF AN OBJECT THROWN WITH A VERY HIGH SPEED FROM A CELESTIAL BODY

Within the frame of our approach, it is interesting to analyze how, an object thrown with a very high speed, from a celestial body, behaves. (Recall that our approach anyway reduces to the classical Newtonian approach, when the speed of the object of concern is negligible as compared to that of light.)

Thus let  $v_{0\text{initial}}$  the launch speed (measured by the local observer), of an object thrown say, vertically, from a celestial body of mass  $M_0$ , and radius  $R_0$ . Its speed shall be  $v_0(r_0)$  (or in

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<sup>§</sup> Special thanks are due to *Professor Elman Hasanov*, from Işık Univesity, who kindly achieved the integrals in question.

short,  $v_0$ ) at the elevation  $r_0$ . In such a case (presuming that  $M_0$  holds as the mass of the gravitational source, throughout), Eq.(13), of Part I, shall be rearranged to be read as

$$\frac{e^{-\alpha_0(R_0)}}{\sqrt{1 - \frac{v_{0\text{initial}}^2}{c_0^2}}} = \frac{e^{-\alpha_0(r_0)}}{\sqrt{1 - \frac{v_0^2(r_0)}{c_0^2}}}. \quad (33)$$

Let further

$$v_0 \cong c_0 \Rightarrow \frac{v_0}{c_0} = 1 - \varepsilon, \quad (34)$$

where  $\varepsilon$  is a *very small number*.

Thus  $v_0(r_0)$  becomes

$$\frac{v_0^2}{c_0^2} \cong 1 - 2\varepsilon e^{-[\alpha_0(R_0) - \alpha_0(r_0)]}. \quad (35)$$

The *initial value* of the LHS of this relationship is  $(1 - 2\varepsilon)$ ; it *rapidly* reduces to the value of  $[1 - 2\varepsilon e^{2\alpha_0(R_0)}]$ .

Henceforth, an object thrown from a celestial body with a speed close to that of *light*, decelerates *exponentially*, acquiring rapidly a *constant speed*.

Consider such a process. The *time* taken for this, is the time for the object to come from  $R_0$  to  $r_0$ , located "*far enough*". The quantity  $\alpha_0(r_0)$ , following Eq.(10) of Part I, is inversely proportional to  $r_0$ ; therefore say,

$$r_0 = 10^6 R_0, \quad (36)$$

can be considered as a *sufficiently long range*, through which the *ultimate escape velocity* is reached, since at the exponential argument of the RHS of Eq.(33),  $\alpha_0(r_0)$  then would become  $10^{-6} \alpha_0(R_0)$  [and thus can be neglected as compared to  $\alpha_0(R_0)$ ].

## 6. CONCLUSION

Herein we presented a *complete theory* covering a *whole range* between the *subatomic world*, and the *world of the motion of stars*. We were able to obtain the results predicted by the *general theory of relativity*, via the *special theory of relativity*, and *quantum mechanics*, only. Note that the exponent "*two*", appearing in the *spatial dependency* of the expression of the *gravitational force* [thus, proportional to the inverse of (*the distance between two static masses*)<sup>2</sup>], can be obtained as a requirement imposed by the *special theory of relativity*.<sup>2,6</sup>

Therefore we did not need the *principle of equivalence*, the *main ingredient of the general theory of relativity*, next to the *special theory of relativity*.<sup>1</sup> This opens, a *much wider horizon*, than that covered by the general theory of relativity. Thus the *gravitational field* is not an *exception* in regards to say, the retardation of *clocks* (*though, it still remains to be the only field interacting with all known objects*). An *ionized clock* interacting with an *electric field*, too will retard, just in the same way a clock interacting with a gravitational field retards.<sup>2,6</sup>

Henceforth, herein, we came out with a *general equation of motion*, governed by a celestial body (*as viewed by the local observer*):

$$-\frac{GM_0}{r_0^2} \left(1 - \frac{v_0^2}{c_0^2}\right) = v_0 \frac{dv_0}{dr_0} . \quad (1)$$

(written by the author, along  $r_0$  and  $v_0$  to be measured by the local observer)

This equation, as viewed by the *distant observer*, becomes

$$-\frac{GM_0}{r^2} e^{-\alpha_0} \left(1 - 2 \frac{v^2}{c_0^2} e^{2\alpha_0}\right) = v \frac{dv}{dr} ; \quad (13)$$

(written by the author, along  $r$  and  $v$  measured by the distant observer)

this turned out to be the equation responsible of the *precession of the perihelion of Mercury*, and any such object.

Our derivation remains as well valid for *any mass in motion*; thus it should also hold, for a *light photon*, if this ever involved a *mass kernel*.

This indeed turned out to be the case, since the *prediction about the deflection of light*, nearby the sun, based on the adoption of such a kernel (*in fact as imposed by our approach, if we were not to endorse any exception, even if this were a photon*), matches well with the observed value.

Thence, one *major conclusion* we come out with, is that the *light photon bears* (*though infinitesimal*), a “*mass kernel*”, a prediction against to the frame of the general theory of relativity.

The way we arrived to our predictions, may further provide a clue to how the *gravitational force* is developed.

Whether this is a *light photon*, *Mercury*, or a *stone*, say in a free fall; we have (*assuming that the counterpart mass of the gravitational force is very big, so that its position is practically not influenced by the gravitational interaction in question*), indeed elucidated the *fact* that, as the (*small*) object accelerates in the gravitational field, a *minimal part* of its *mass*, or the same,

“*internal energy*”, is transformed into *kinetic energy*; as the object decelerates, it piles up “*kinetic energy*” as *additional mass*.

It is as if energy “*condenses*” into “*mass*”, or mass “*sublimes*” into “*translational energy*”, or some other kind of energy (*a process well known by nuclear engineers, as well as say, accelerator researchers, but seemingly overlooked regarding gravitation*).

It is further very interesting to note that, based on Eq.(1), an object thrown from a celestial body with a speed close to that of light, decelerates exponentially, thus to reach very fast a constant speed.

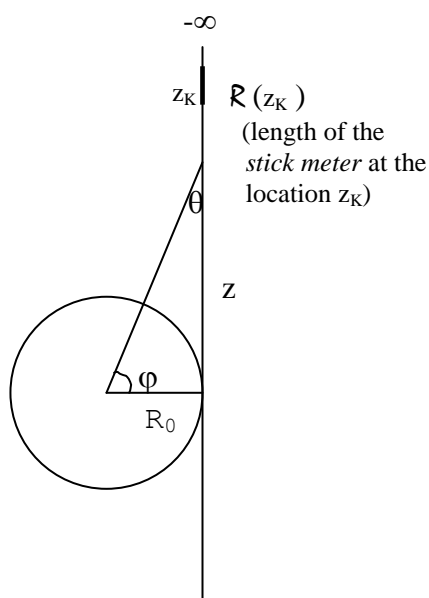
How come that; some of our results, such as the *retardation of clocks in a gravitational field*, are practically *the same as those of Einstein*; some of them, such as the *precession of the perihelion of Mercury*, are *approximately, or in the specific case of Mercury virtually the same as those of Einstein*; and some of them, such as the *prediction of no black holes, the stretching of a length, the decrease of mass, in a gravitational field, and even more fundamentally, the divergence of the inertial and gravitational masses, are very different than the basic elements of the frame forwarded by Einstein, is a whole different story, needing to be examined, throughout a separate work.*

Nonetheless as evoked by Eq.(15), it seems that *assuming the equality of the inertial mass and the gravitational mass, and overlooking the mass equivalence of the gravitational energy, constitute effects of about the same magnitude and amazingly canceling each other; this should be how we could reproduce virtually the same result as that of Einstein, in regards to the precession of the perihelion of Mercury. This occurrence we believe, does not reduce the unequal wholeness of our approach.*

At any rate Einstein’s approach leads to the fact that, the “*mass-energy relationship*” of special theory of relativity, the fundamental ingredient to the general theory of relativity, does not hold, for values of the *couple of mass and energy, belonging to different sets of gravitational coordinates.*

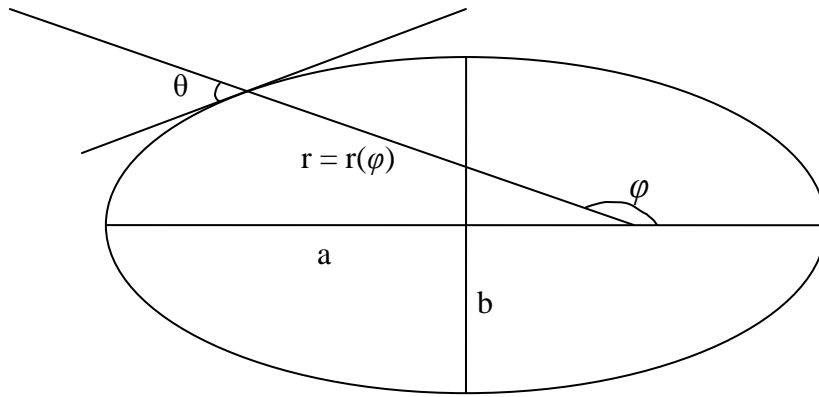
One can further check that the dimension of Planck Constant, though an invariant within the frame of the special theory of relativity, is not preserved, through the general theory of relativity. This too, should be an essential alarm.

Our approach is free of such annoyances.





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**Figure 2** Mercury's orbit

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## REFERENCES

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- <sup>1</sup> A. Einstein, **The Meaning of Relativity**, Princeton University Press, 1953.
- <sup>2</sup> T. Yarman, **A Novel Approach to The End Results of The General Theory of Relativity Via Just The Special Theory of Relativity and Quantum Mechanics**, Communication to Professor C. Marchal, ONERA (May 1999), also to Professor N. Veziroğlu, University of Miami (December 2001), as well as to Professor V. Rozanov, Lebedev Institute (January 2002).
- <sup>3</sup> H. C. Ohanian, **Gravitation and SpaceTime**, W. W. Norton & Company, Inc., 1976.
- <sup>4</sup> R. V. Pond, G. A. Rebka, Jr., *Physical Review Letters*, 4, 337 (1960).
- <sup>5</sup> I. Shapiro, Gordon H. Pettengill et al., **Mercury's Perihilion Advence: Determination by Radar**, *Physical Review Letters*, Volume 28, Number 24, 1972.
- <sup>6</sup> T. Yarman, **A Novel Approach to The End Results of The General Theory of Relativity and to Bound Muon Decay Retardation**, DAMOP 2001 Meeting, APS, May 16-19, 2001, London, Ontario, Canada.