THE GENERAL EQUATION OF MOTION VIA THE SPECIAL THEORY OF RELATIVITY AND QUANTUM MECHANICS PART I: A NEW APPROACH TO NEWTON EQUATION OF MOTION

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#### Abstract

Herein we present a whole new approach to the derivation of the Newton Equation of Motion; throughout Part II of the present work, this shall lead to the findings brought up within the frame of the general theory of relativity (such as the precession of the perihelion of the planets, and the deflection of light nearby a star). To the contrary of what had been generally achieved so far, our basis consists in supposing that the gravitational field, through the binding process, alters the "rest mass" of an object conveyed in it. In fact, the special theory of relativity already imposes such a change. Next to this fundamental theory, we use the classical Newtonian gravitational attraction, reigning between two static masses. We have previously shown however that the $1 / r^{2}$ dependency of the gravitational force is also imposed by the special theory of relativity.

Our metric, is (just like the one used by the general theory of relativity) altered by the gravitational field (in fact, by any field the "measurement unit" in hand interacts with); yet in our approach, this occurs via quantum mechanics. More specifically, the rest mass of an object in a gravitational field is decreased as much as its binding energy in the field. A mass deficiency conversely, via quantum mechanics, yields the stretching of the size of the object in hand, as well as the weakening of its internal energy. Henceforth we shall not need the "principle of equivalence" assumed by the general theory of relativity, in order to predict the occurrences dealt with this theory.


Thus we start with the following interesting postulate, in fact nothing else but the conservation of energy, in the broader relativistic sense of the concept of "energy".

Postulate: The rest mass of an object bound to a celestial body amounts less than its rest mass measured in empty space, and this as much as its binding energy vis-à-vis the gravitational field of concern.

This yields (with the familiar notation), the interesting equation of motion driven by the celestial body of concern, i.e.

$$
\frac{\mathrm{e}^{-\alpha_{0}\left(\mathrm{r}_{0}\right)}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}=\text { Cons } \tan \mathrm{t} ; \alpha_{0}\left(\mathrm{r}_{0}\right)=\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0} \mathrm{c}_{0}^{2}} ;
$$

here $M_{0}$ is the mass of the celestial body creating the gravitational field of concern; $G$ is the universal gravitational constant; $\mathrm{r}_{0}$ points to a location picked up on the trajectory of the motion; $\mathrm{v}_{0}$ is the tangential velocity of the object at $\mathrm{r}_{0}$, and $\mathrm{c}_{0}$ the speed of light in empty space.

The above relationship tells us that the mass of the object in motion can be conceived as made of its mass brought from infinity, at the location defined by $\mathrm{r}_{0}$ on its trajectory, thus i) decreased as much as its binding energy, ii) but at the same time, increased by a Lorentz factor, due to its translational motion on the trajectory.

The differentiation of this relationship leads to

$$
-\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0}^{2}}\left(1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}\right)=\mathrm{v}_{0} \frac{\mathrm{dv}_{0}}{\mathrm{dr}_{0}} .
$$

This differential equation is the classical Newton Equation of Motion, were $\mathrm{v}_{0}$, negligible as compared to $\mathrm{c}_{0}$ (the speed of light in empty space).

## 1. INTRODUCTION

Herein we present a whole new approach to the derivation of the Newton Equation of Motion as well as the findings brought up within the frame of the general theory of relativity (such as the "precession of the perihelion of the planets", and the "deflection of light nearby a star").

To the contrary of what had been generally achieved so far, our basis consists in supposing that the gravitational field, through the binding process, alters the "rest mass" of an object conveyed in it. In fact, the special theory of relativity, though astonishingly far and wide overlooked, imposes such a change. Next to this fundamental theory, we use the classical Newtonian gravitational attraction reigning between two static masses. We have previously shown however that the $1 / r^{2}$ dependency of the gravitational force is also imposed by the special theory of relativity. ${ }^{1}$

Furthermore our metric, is (just like the one used by the general theory of relativity) altered by the gravitational field (in fact, by any field the "measurement unit" in hand interacts with); yet in our approach, this occurs via quantum mechanics. In effect, the solution of even a nonrelativistic quantum mechanical description, given that "potential energies existing in nature" are considered, bears a casing, in perfect harmony with the special theory of relativity. This is to say, regarding the internal dynamics of a wave-like object, "space" (i.e. the size of the object), "time" (period of the internal dynamics of concern), and "mass" (the mass, to be associated with the wave-like object, working as the "pendulum mass" of its internal dynamics), are structured in such a way that their interrelation remains Lorentz invariant (i.e. invariant, were the object brought into a uniform translational motion).

Thus, as we shall see, the rest mass of an object in a gravitational field is decreased as much as its binding energy in the field.

A mass deficiency conversely, via quantum mechanics, yields a stretching of its size, as well as the weakening of its internal energy. This is how our metric is altered by the field.

Therefore the basis of our approach shrinks down to only the special theory of relativity.
Henceforth we shall not need the "principle of equivalence" assumed by the general theory of relativity, in order to predict the occurrences dealt with this theory. ${ }^{2}$ We predict them through the general equation of motion we shall establish herein (thus, essentially based on the special theory of relativity, only).

A change, through the binding process, in the rest mass of an object interacting with a gravitational field, though, seems somewhat clear. Indeed, the special theory of relativity predicts such an occurrence as, for example, the proton and the electron, when bound to each other in the hydrogen atom, weigh less than the sum of the proton and the electron, carried away from each other; the mass deficiency in question is (by taking the speed of light, unity), exactly equal to the binding energy of the proton and the electron in the hydrogen atom, i.e. 13.6 ev , based on the fundamental relationship ${ }^{3}$
$($ Energy released, or acquired $)=($ Magnitude of the algebraic increase in the mass $)$ $x$ (Speed of light in empty space) ${ }^{2}$.

So, contrary to the widespread opinion, the electron or the proton cannot be the same, when bound to each other; they are different. Their internal dynamics altogether, weaken as much as 13.6 ev , when they are bound to each other to shape up the hydrogen atom.

Many scientists though, still firmly think that there is the "proper mass" (rest mass) and the "relativistic mass" (defined within the frame of the special theory of relativity), and that the proper mass is, whatsoever an invarian, $t$ which is a characteristic of matter, and that is all.

Generally speaking, this is unacceptable. The proper mass of a given particle on the whole at rest may, depending on the circumstances, embody a more or less energetic internal motion; this will, one way or the other, affect the proper mass.

Suppose indeed that Captain Electron (we mean, the electron itself) is cruising in a full electric isolation, with a uniform translational velocity. So does Captain Proton (i.e. the proton itself). They approach to each other. Then (based on the special theory of relativity) we would be certain that, Captain Electron in its own frame of reference, all the way through, preserves its identity, defined at infinity. (So will also do Captain Proton.) If now, we remove the previous electric isolation, Captain Electron and Captain Proton, because of the electric attraction force, they mutually create, shall start getting accelerated toward each other. The "extra kinetic energy" they would acquire through this process, shall be supplied by the system made of the two. Their total energy [i.e. (the sum of their relativistic masses) x (the speed of light $)^{2}$ ], through the motion, shall remain constant, and equal to the equivalent of the sum of their initial relativistic masses. (Otherwise, the energy conservation law would be broken.) Let us suppose for simplicity that in the latter case (where we have no electric isolation), they start, far away from each other, at rest; then their initial relativistic masses are, essentially, identical to respectively their rest masses. If now the accelerating Captain Electron, say in Captain Proton's frame of reference, hurts an obstacle and looses all the kinetic energy, it would have acquired through the attraction process; thence, it must concurrently dump a portion of its rest mass, and this, as much as the amount of the kinetic energy it would have piled up, on the way.

Thus we cannot say that the proton and the electron are the same after we have retrieved from the system made of the two, a given amount of energy, no matter how much. The greater is the energy extracted, the harder will be the harm caused in their internal dynamics, thus in their proper masses defined at infinity.

This is exactly what happens when, say the hydrogen atom is formed, except that the electron, as referenced to the proton is not anymore at rest, but possesses a given amount of kinetic energy; still an energy of 13.6 ev is needed, to carry the electron away from the proton, back to infinity.

It is thus clear that as referenced to the proton, or (since the proton is much too big as compared to the electron), practically the same, as referenced to the laboratory system, the hydrogen electron's proper (rest) mass, is altered as much.

Just the same way, the daily production of thermal energy, is due to the transformation of a minimal part of the mass entering in reaction, into energy. Thus the reaction products weigh less than the reactants, and this, as much as the energy produced throughout.

The fuel, i.e. coal, petroleum, uranium, plutonium, anything, in a power plant of, say $3000 \mathrm{MW}_{\text {thermal }}$, continuously working for a period of one year, thus producing an energy amounting to $3000 \mathrm{MW}_{\text {thermal }} \mathrm{x}$ year, at the end of this period, weighs less, as much as the equivalent of the energy output in question, i.e. [based on the equivalence between mass and energy], about 1 kg . This is of course insignificant as compared to millions of tons of coal or petroleum that would be fired into the plant of concern, but well detectable as compared to about a ton of plutonium-239, or uranium-235 needed to be depleted in a nuclear power plant of $3000 \mathrm{MW}_{\text {thermal }}$ through a period of one year.

In the same way, a compressed spring should be heavier than the same spring when stretched out; or the gas in a room at a high temperature should weigh more than the "same gas" at a lower temperature, etc.

All these, already happen to be well established facts. Thus, any proper mass weighs less, after releasing energy, or conversely it shall weigh more, after piling up energy.

Therefore, herein we anticipate that when an object is bound to a celestial body, its rest mass (measured in empty space) is decreased as much as the binding energy, it would have developed in the gravitational field of the celestial body of concern. ${ }^{4}$

Einstein in his general theory of relativity, considers the conservation of the "rest masses", instead of the conservation of the "total energy". ${ }^{2}$

Yılmaz somewhat fulfilled this gap; thus he derived the "exact solution" of the "accelerated elevator", and to his great surprise, found out that Einstein's fields equations were not satisfied; this was the beginning of Yılmaz's efforts towards a more consistent theory, though along the same direction as that drawn by Einstein. ${ }^{5,6,7}$

At any rate, Einstein's general theory of relativity leads to the fact that, his original relativistic "mass-energy relationship" [i.e. Eq.(1)], does not hold between gravitational coordinate values of energy and mass, in any perceptible way. ${ }^{8}$ We think that this is a fallacy. We do not have such an annoyance, since we derived our results essentially based on Einstein's "massenergy relationship" obtained within the frame of the special theory of relativity.

Thence we state our main postulate, in fact nothing else, but the energy conservation law, where though, as introduced by the special theory of relativity, [energy] and [(mass) x (speed of light $)^{2}$ ] [cf. Eq.(1)], are no different from each other.

Postulate: The rest mass of an object bound to a celestial body amounts less than its rest mass measured in empty space, and this as much as its binding energy vis-à-vis the gravitational field of concern.

It is important to note that, on the contrary to what the general theory of relativity eventually formulates, as we shall see here it is question of a decrease of mass in a gravitational field, and this is interestingly, just as much as the mass increase (due to acceleration) formulated by the former theory. ${ }^{1}$

Note further that, so far there had been no measurement of mass in a gravitational field; thus a measurement of mass at two different altitudes on Earth, can furnish a verification of our guess.

According to recent developments ${ }^{9,10}$ this indeed seems possible, if one proposes to achieve the measurement of mass through the measurement of the Rydberg Constant ${ }^{*}$, already measured with a precision of $10^{-14}$, whereas a precision of $10^{-13}$ is good enough for a difference of altitudes of $10^{3} \mathrm{~m}$.

Below, we first sketch how the gravitational binding energy reduces the rest mass of an object bound to the celestial body in consideration (Section 2). Then, we recall the quantum mechanical theorems we have established previously (Section 3) (though we will particularly need them, throughout Part II of our work). An elaboration on the gravitational binding energy follows (Section 4). The change of the rest mass of an object in a gravitational field, together with the Lorentz mass dilation, due to the local motion, yields the general equation of motion (Sections 5 and 6). A conclusion follows (Section 7).

Next, taking into account how unit lengths, quantum mechanically stretch in a gravitational field, we shall be able to obtain the precession of the perihilion of Mercury (or anything as such), as well as the deflection of light grazing a celestial body; this shall constitute the content of Part II of our work.

## 2. THE GRAVITATIONAL BINDING ENERGY

At this stage, we have to evaluate the gravitational binding energy. For this purpose naturally, we have to use the expression for the gravitational force.

Herein we consider only the gravitational force between two static masses.
Still, since we aim ultimately at deriving a result obtained within the context of the general theory of relativity, without having to rely on it, we better should not plainly borrow, the

[^0]expression for the gravitational force (between two static masses), with its classical empirical form, from Newton ${ }^{11}$, since this (were the case that of a weak gravitational field), is formally, well manufactured by the general theory of relativity.

Therefore (and luckily) we derive the $1 / r^{2}$ dependency of the gravitational force between two static masses, here again, from the special theory of relativity. ${ }^{1}$

Hence, we can calculate the binding energy $\mathrm{E}_{\mathrm{B}}$, of a given object in the gravitational field of the celestial body of concern, in the usual way. As a first approximation, let us consider that the binding energy is small as compared to the mass of the object. Thus:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}}=\int_{\mathrm{R}_{0}}^{\infty} \mathrm{G} \frac{\mathrm{M}_{0} \mathrm{~m}_{0}}{\mathrm{r}^{2}} \mathrm{dr} \cong \mathrm{G} \frac{\mathrm{M}_{0} \mathrm{~m}_{0}}{\mathrm{R}_{0}} \tag{2}
\end{equation*}
$$

here $M_{0}$ is the mass of the host body binding the object of mass $\mathrm{m}_{0}$, as measured in empty space, $\mathrm{R}_{0}$ the distance of the mass $\mathrm{m}_{0}$ to the center of the host body, and G the universal gravitational constant.

In reality $\mathrm{m}_{0}$ (based on the discussion we presented above, in Section 1), changes continuously throughout. One can, as we shall soon see, easily elaborate on this.

When the object of mass $\mathrm{m}_{0}$ is bound to the gravitational field, $\mathrm{m}_{0}$ decreases to become m , such that

$$
\begin{equation*}
\mathrm{m}_{0} \rightarrow \mathrm{~m}, \mathrm{~m}=\chi \mathrm{m}_{0} \tag{3}
\end{equation*}
$$

$\chi$ is determined out of Eq.(1), more specifically

$$
\begin{equation*}
\left(\mathrm{m}_{0}-\mathrm{m}\right) \mathrm{c}_{0}^{2}=\mathrm{E}_{\mathrm{B}} . \tag{4}
\end{equation*}
$$

Given $\mathrm{E}_{\mathrm{B}}, \chi($ smaller than unity) becomes:

$$
\begin{equation*}
\chi=1-\frac{\mathrm{E}_{\mathrm{B}}}{\mathrm{~m}_{0} \mathrm{c}_{0}^{2}}=1-\frac{\mathrm{GM}_{0}}{\mathrm{R}_{0} \mathrm{c}_{0}^{2}} . \tag{5}
\end{equation*}
$$

## 3. THEOREMS WE HAVE ESTABLISHED PREVIOUSLY

Our approach becomes very interesting, if one recalls the following theorem we have proven elsewhere. ${ }^{12,13,14,15,16,17}$

Theorem 1: In a "real wave-like description" (thus, not embodying artificial potential energies), composed of I electrons and J nuclei, if the (identical) electron masses $m_{i 0}, i=1, \ldots, I$ and different nuclei masses $m_{j 0}, j=1, \ldots, J$, involved by the object, are overall multiplied by the arbitrary number $\chi$, then concurrently, a) the total energy $\mathrm{E}_{0}$ associated with the given clock's motion of the object, is increased as much, or the same, the period $\mathrm{T}_{0}$, of the motion associated with this energy, is decreased as much, and b) the characteristic
length or the size $\mathrm{R}_{0}$ to be associated with the given clock's motion of the object, contracts as much; in mathematical words this is

$$
\begin{align*}
& \left\{\left[\left(\mathrm{m}_{\mathrm{i} 0}, \mathrm{i}=1, \ldots, \mathrm{I}\right) \rightarrow\left(\chi \mathrm{m}_{\mathrm{i} 0}, \mathrm{i}=1, \ldots, \mathrm{I}\right)\right],\left[\left(\mathrm{m}_{\mathrm{j} 0}, \mathrm{j}=1, \ldots, \mathrm{~J}\right) \rightarrow\left(\chi \mathrm{m}_{\mathrm{j} 0}, \mathrm{j}=1, \ldots, \mathrm{~J}\right)\right]\right\} \\
& \quad \Rightarrow\left\{\left[\mathrm{E}_{0} \rightarrow \chi \mathrm{E}_{0}\right],\left[\mathrm{T}_{0} \rightarrow \frac{\mathrm{~T}_{0}}{\chi}\right],\left[\mathrm{R}_{0} \rightarrow \frac{\mathrm{R}_{0}}{\chi}\right]\right\} \tag{6}
\end{align*}
$$

Then, following the above derivation, we come at once, to the next theorem.

Theorem 2: A wave-like clock in a gravitational field, retards via quantum mechanics, due to the mass deficiency it develops in there, and this, as much as the binding energy it displays in the gravitational field; at the same time and for the same reason, the space size in which it is installed, stretches as much.

This can further be grasped rather easily as follows. The mass deficiency the wave-like object displays in the gravitational field weakens its internal dynamics as much. Thence, we arrive at the two principal results, we just stated.

Note that, according to our approach, the classical gravitational redshift and a related mass decrease, occur to be concomitant quantum mechanical effects. Thus in fact, contrary to what the general theory of relativity ultimately predicts, we expect a mass decrease in a gravitational field (and not a mass increase).

It is of course impressive to notice that the foregoing reasoning is not restricted to gravitation only. It should hold in any kind of interaction where the wave-like clock, develops a binding, thus undertakes a mass deficiency (without of course, loosing its identity), as described above; in such a case, $\mathrm{E}_{\mathrm{B}}$ becomes the binding energy of the wave-like clock to either field (electric, magnetic, nuclear, gravitational, whatever) of concern, our finding holds. ${ }^{1}$ So, quite on the contrary to the prevailing opinion, the gravitational field is not any different than other fields in affecting the clocks. Thus, we could establish the following simple theorem, generalizing the previous one. ${ }^{12}$

Theorem 3: A wave-like clock interacting with any field, electric, nuclear, gravitational, or else (without loosing its identity), retards as much as its binding energy, developed in this field.

The foregoing Theorems 1 and 2 will be specifically used in Part II of our work.
Let us now elaborate on the binding energy.

## 4. ELABORATION ON THE GRAVITATIONAL BINDING ENERGY

In calculating the binding energy $\mathrm{E}_{\mathrm{B}}$, at the level of Eq.(2), we had tacitly assumed that the wave-like clock of original mass $\mathrm{m}_{0}$, looses only an insignificant part of it, through the binding process. Otherwise, Eq.(7) should be written as follows: ${ }^{1}$

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}}(\mathrm{r})=\mathrm{GM}_{0} \int_{\mathrm{r}}^{\infty} \frac{\mathrm{m}_{0} \mathrm{c}^{2}-\mathrm{E}_{\mathrm{B}}\left(\mathrm{r}^{\prime}\right)}{\mathrm{r}^{\prime 2} \mathrm{c}^{2}} \mathrm{dr}^{\prime}, \tag{7}
\end{equation*}
$$

which leads to the differential equation

$$
\begin{equation*}
-\frac{\mathrm{dE}_{\mathrm{B}}(\mathrm{r})}{\mathrm{dr}}+\frac{\mathrm{GM}_{0} \mathrm{E}_{\mathrm{B}}(\mathrm{r})}{\mathrm{r}^{2} \mathrm{c}^{2}}=\frac{\mathrm{GM}_{0} \mathrm{~m}_{0}}{\mathrm{r}^{2}}, \tag{8}
\end{equation*}
$$

and finally to

$$
\begin{equation*}
E_{B}\left(R_{0}\right)=m_{0} c^{2}\left[1-\exp \left(-\frac{G M_{0}}{\mathrm{c}^{2} R_{0}}\right)\right]=m_{0} c^{2}\left\lfloor 1-e^{-\alpha\left(R_{0}\right)}\right\rfloor, \tag{9}
\end{equation*}
$$

at a distance $\mathrm{R}_{0}$ from the center of the host celestial mass $M_{0}$, via the usual definition

$$
\begin{equation*}
\alpha=\alpha(r)=\frac{G M_{0}}{\mathrm{c}^{2} \mathrm{r}} \tag{10}
\end{equation*}
$$

The outcome $\mathrm{E}_{\mathrm{B}}$ of Eq.(9) is zero when $\mathrm{m}_{0}$ is at infinity; $\mathrm{E}_{\mathrm{B}}$ becomes more and more important as $\alpha$ increases. Yet here, there appears to be no singularity at all (unless $\mathrm{m}_{0}$ when transplanted nearby $M_{0}$, is somehow degenerated). This seems to be quite remarkable, since (based on our Theorems 1 and 2) it yields no singularity in time, thus no "black holes".

Note that Eq.(9), along Eqs. (3) and (5), leads to

$$
\begin{equation*}
\mathrm{m}(\mathrm{r})=\mathrm{m}_{0} \mathrm{e}^{-\alpha} \tag{11}
\end{equation*}
$$

We would like to say few words about how, we come to a mass decrease in a gravitational field, instead of a concluding mass increase predicted by the general theory of relativity.

Einstein's depart point is, based on the equivalence principle, a mass increase displayed by the object, carried away by the "accelerating elevator". This depart point, though a striking idea, seems inappropriate for (chiefly, next to the reason we will develop herein) one major reason; it is that, there is a clear asymmetry between the accelerating elevator and the gravitational field, with respect to a distant observer. "Getting on the accelerating elevator" (when we are nearby at rest, in empty space) and "getting on a celestial body" (from empty space), indeed are not at all the same process (for the distant observer), clearly at least for one thing, i.e. he has to get accelerated to be able to catch up with the accelerating elevator, whereas he has to get decelerated in order to be able to land on the celestial body. The first process as expected (within the context of the theory of relativity) yields a mass increase, whereas the second one, as we saw, gives a mass decrease (with respect to the distant observer).

A mass decrease, through our Theorem 2, yields a unit time increase, but also a length loosening (not a length contraction).

Thence, Einstein's transposition, of mass increase and a concurrent length contraction taking place in an accelerating elevator, to a gravitational field, seems to be incorrect. We shall elaborate further, on this, below.

Nonetheless our Eqs. (2), (9) and (11), to a certain extent happen to be in agreement ${ }^{\dagger}$ with the gravitational potential furnished by the general theory of relativity. ${ }^{18}$

How come?
As we shall detail below, briefly for one thing it seems that, assuming the equality of the inertial mass and the gravitational mass, and overlooking the mass equivalence of the gravitational energy, constitute effects of about the same magnitude and amazingly canceling each other; this should be how we could reproduce practically the same result as that of Einstein, in regards to the gravitational potential, the precession of the perihelion of Mercury (that we shall elaborate in Part II), etc. Recall anyway that even alike predictions made by the general theory of relativity and the theory presented herein, are not exactly the same.

## 5. THE GENERAL EQUATION OF GRAVITATIONAL MOTION IN SCALAR FORM

Now, we are ready to derive the general equation of gravitational motion.
The idea behind it is strikingly simple and is rooted to our postulate, stated above. When an object enters into interaction with a celestial body, its "total energy" (as conceived within the frame of the special theory of relativity), throughout, remains the same. The extra kinetic energy it shall acquire or it shall lose on the way, shall be accounted by an equivalent change in its rest mass.

[^1]\[

$$
\begin{equation*}
\mathrm{V}(\mathrm{r})=-\frac{\mathrm{GM}}{0} \mathrm{r}+\frac{\mathrm{G}^{2} \mathrm{M}_{0}^{2}}{\mathrm{r}^{2} \mathrm{c}_{0}^{2}} \quad \text { (furnished by the classical general theory of relativity), } \tag{i}
\end{equation*}
$$

\]

whereas Eq.(2), together with Eq.(5), furnishes

$$
\begin{equation*}
V(r)=-\frac{E_{B}}{m_{0}}=-\frac{G M_{0} \gamma}{r}=-\frac{G M_{0}}{r}\left(1-\frac{G M_{0}}{r}\right)=-\frac{G M_{0}}{r}+\frac{G^{2} M_{0}^{2}}{r^{2} c_{0}^{2}} . \tag{ii}
\end{equation*}
$$

(furnished, within the given approximation, by the theory presented herein) (c.q.f.d.)

Note that the above expression can further be elaborated by letting the mass of the object of concern vary under the integral operation in Eq.(2). The resulting binding energy $E_{B}$, turns out to be the RHS of Eq.(16), presented above; accordingly the gravitational potential $\mathrm{V}(\mathrm{r})$ becomes

$$
\begin{equation*}
\mathrm{V}(\mathrm{r})=-\frac{\mathrm{E}_{\mathrm{B}}}{\mathrm{~m}_{0}}=-\frac{\mathrm{GM} M_{0}}{\mathrm{r}}+\frac{1}{2} \frac{\mathrm{G}^{2} M_{0}^{2}}{\mathrm{r}^{2} \mathrm{c}_{0}^{2}}-\frac{1}{6} \frac{\mathrm{G}^{3} M_{0}^{3}}{\mathrm{r}^{3} \mathrm{c}_{0}^{4}} \quad \text { (rigorously furnished by the theory presented herein). } \tag{iii}
\end{equation*}
$$

Henceforth, when an object falling in a gravitational field, is stopped and the kinetic energy, it would have acquired is taken away, its rest mass (as measured in empty space) should be decreased as much as the binding energy it would have developed in the field.

Here, to make things easier, we tacitly assumed that, one of the interacting objects is very massive, and the other is relatively very small, so that we have to worry about only the small one. The one which is massive undergoes practically no change. Our approach can be easily extended to the general case.

In order to ease our dissertation we shall work on a concrete basis, more specifically we will consider the planet Mercury, in motion around the sun (without though any loss of generality).

We can conceive Mercury's motion (around the sun), as made of two steps:
i) Bring it from infinity to a distance $r_{0}$ from the sun, to a given location on its "elliptical" orbit; the energy this process requires, is the classical potential energy.
ii) Deliver to it, the kinetic energy it would display on this location. (Note that on the orbit, the classical total energy, i.e. potential energy + kinetic energy, is a constant of the motion.)

Let us then make the following casual definitions.
$\mathrm{r}_{0}$ or $\mathrm{r}_{0}\left(\mathrm{t}_{0}\right) \quad:$ distance of the sun to the planet, at time $\mathrm{t}_{0}$ (as referred to the local observer)
$\mathrm{m}_{0 \infty} \quad:$ the planet's rest mass at infinity
$\mathrm{m}_{0}\left(\mathrm{r}_{0}\right)$ or $\mathrm{m}_{0}\left(\mathrm{t}_{0}\right) \quad$ : the planet's rest mass at a distance $\mathrm{r}_{0}$, or at the corresponding time $\mathrm{t}_{0}$, as referred to the sun
$\mathrm{m}_{0 \gamma}\left(\mathrm{r}_{0}\right)$ or $\mathrm{m}_{0 \gamma}\left(\mathrm{t}_{0}\right) \quad$ : the planet's total relativistic mass (which is its mass at infinity decreased as much as its binding energy, and increased based on the special theory of relativity, due to its "translational" motion on the orbit), at $\mathrm{r}_{0}$, and at the corresponding time $\mathrm{t}_{0}$
$\mathrm{v}_{0}$ or $\mathrm{v}_{0}\left(\mathrm{t}_{0}\right)$ or $\mathrm{v}_{0}\left(\mathrm{r}_{0}\right)$ : magnitude of the tangential velocity of the planet on the orbit, at the location $r_{0}$, and at the corresponding time $t_{0}$
$\mathrm{c}_{0} \quad$ : the velocity of light in empty space (free of any gravitational field)
$\alpha_{0}$ or $\alpha_{0}\left(r_{0}\right) \quad:$ dimensionless quantity defined along Eq.(10), for the distance $r_{0}$ of the planet from the sun

## Equation of Motion of Mercury as Assessed by the Local Observer

Starting with our energy conservation postulate, and the above definitions; as assessed by the local observer, we can now write the following equations based on, first, Eqs. (10) and (11), yielding decrease of the rest mass of the planet at infinity, and then, the familiar relativistic mass increase with tangential velocity on the orbit:

$$
\begin{equation*}
\mathrm{m}_{0}\left(\mathrm{r}_{0}\right)=\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}\left(\mathrm{r}_{0}\right)}, \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\mathrm{m}_{0 \gamma}\left(\mathrm{r}_{0}\right)}{\mathrm{m}_{0 \infty}}=\frac{\mathrm{e}^{-\alpha_{0}\left(\mathrm{r}_{0}\right)}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}=\boldsymbol{D} \tag{13}
\end{equation*}
$$

where $D$ is a constant we are to determine.

Note that $\alpha_{0}\left(\mathrm{r}_{0}\right)$ remains as a constant in the case of a "circular orbit"; so is $\mathrm{v}_{0}\left(\mathrm{r}_{0}\right)$; thus, $D$ is anyhow a constant. This special case, does not advance us. Yet what is interesting, as we propose to study, is that $D$ is whatsoever, a constant.

Let us explain this, a bit further. According to our approach (we stated as a postulate above), $\mathrm{c}_{0}^{2} \mathrm{~m}_{\gamma}\left(\mathrm{r}_{0}\right)$ (the total relativistic energy of the planet) ought to be a constant all along Mercury's journey around the sun. As the planet speeds up nearby the sun, it is that, an infinitesimal part of its mass somewhat "sublimes" to get transformed into kinetic energy, yielding the extra kinetic energy (the planet acquires as it speeds up); as it slows down away from the sun, through its orbital motion, it is that, a portion of its kinetic energy somewhat "condenses" onto its rest mass, on the orbit.

This alternating process through the motion, based on the special theory of relativity, anyway, makes that the planet's total relativistic mass (i.e. the classical mass + the mass equivalent of the kinetic energy) remains the same. This should be considered harmonious with the fact that the planet's classical total energy on the orbit, is constant. We will soon elaborate on this point.

What is this constant? It would first be interesting to examine the case of free fall, where D (as we shall see) is unity.

## Free Fall

Consider an object originally at rest, at "infinity", and experiencing a free fall in a gravitational field. Let $\mathrm{m}_{00}$ its rest mass, at infinity. Its binding energy $\mathrm{E}_{\mathrm{B}}$, were it stopped at a given altitude, according to Eq.(9) is

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}}=\mathrm{m}_{0 \infty} \mathrm{c}_{0}^{2}\left(1-\mathrm{e}^{-\alpha_{0}}\right), \tag{14}
\end{equation*}
$$

where $\alpha_{0}$ represents the value of this quantity at the altitude in consideration.
The rest mass of the object at this altitude, according to Eq.(11), is $\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}}$.
On the other hand, the object through its free fall, would (up to the altitude of concern) acquire the velocity $\mathrm{v}_{0}$, yielding the relativistic mass $\mathrm{m}_{0 \mathrm{e}} \mathrm{e}^{-\alpha_{0}} / \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$, while some of its mass content, as just mentioned, is transformed into kinetic energy. The differrence of the corresponding energies is nothing, but the binding energy $\mathrm{E}_{\mathrm{B}}$ [given by Eq.(14)]:

$$
\begin{equation*}
\mathrm{m}_{0 \infty}\left(1-\mathrm{e}^{-\alpha_{0}}\right)=\frac{\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}-\mathrm{m}_{0 \infty} \mathrm{e}^{-\mathrm{\alpha}_{0}} \tag{15}
\end{equation*}
$$

This, right away yields unity, for the constant D, appearing in Eq.(13) i.e.

$$
\begin{equation*}
\frac{\mathrm{e}^{-\alpha_{0}}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}=1 \quad \text { (in the case of an object in free fall, started at rest, at" infinity" ). } \tag{16}
\end{equation*}
$$

Note that the classical total energy, i.e. potential energy + kinetic energy, through the free fall is conserved, which is quite harmonious, with Eq.(15).

If the falling object started at "infinity" with an initial velocity $\mathrm{v}_{0 \infty}$, and not at rest, than Eq.(16) would become

$$
\begin{equation*}
\frac{\mathrm{e}^{-\alpha_{0}}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}=\frac{1}{\sqrt{1-\frac{\mathrm{v}_{0 \infty}^{2}}{\mathrm{c}_{0}^{2}}}} \text { (in the case of an object in free fall, started with the initial velocity } \mathrm{v}_{0 \infty} \text {, at " infinity" ). } \tag{17}
\end{equation*}
$$

Note that no matter what the direction of the initial velocity $\mathrm{v}_{0_{\infty}}$ at infinity, or the direction of $\mathrm{v}_{0}$ at the given location, we associate with the object in hand, the above relationship is still valid.

## Differential Equation of Motion as Assessed by the Local Observer

The constancy of $D$ can further be easily checked and fixed for the case of Mercury, based on the actual data associated with the planet, at a given location of it, on the orbit. ${ }^{*}$

For further simplicity we can recall that the orbit of the planet is nearly circular.
Thus, based on Eq.(13), we can write

$$
\begin{equation*}
\frac{\mathrm{e}^{-\alpha_{0}}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}=\mathrm{D} \cong 1-\alpha_{0}+\frac{1}{2} \frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}} \tag{18}
\end{equation*}
$$

[^2]it can indeed be checked that, at any location on the orbit of Mercury, we precisely have
$$
c_{0}^{2}\left(1-D^{2}\right)=1.15 \times 10^{9} \cdot \mathrm{~km}^{2} / \mathrm{s}^{2}
$$

Here recall that $\mathrm{v}_{0}$ is the tangential velocity of the planet, at the location $\mathrm{r}_{0}$ on the orbit (as referred to the local observer).

Note further that

$$
\begin{equation*}
\alpha_{0}=\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0} \mathrm{c}_{0}^{2}} \cong \frac{\mathrm{GM}_{0}}{\overline{\mathrm{r}_{0}} \mathrm{c}_{0}^{2}}=\overline{\alpha_{0}} \cong \frac{\overline{\mathrm{v}_{0}^{2}}}{\mathrm{c}_{0}^{2}} \cong 2.56 \times 10^{-8}, \tag{19}
\end{equation*}
$$

where (for a reason that we shall clarify right away) we associate with $\mathrm{r}_{0}$, the quantity $\overline{\mathrm{r}_{0}}$, i.e. the average distance of the planet to the sun (which happens to be the semi-grand axis of the elliptical orbit); $\overline{\alpha_{0}}$ is the average of $\alpha_{0}$, and $\overline{v_{0}^{2}}$ the mean square velocity.

It is already striking that the second equality displayed by Eq.(18), under the assumptions in question (i.e. small $\mathrm{v}_{0}$, small $\alpha_{0}$ ), is nothing but, the Newton's equation of gravitational motion (in its integral form), relating the tangential velocity $\mathrm{v}_{0}$ of the planet, to its distance to the sun.

The usual form of the equality of concern is ${ }^{19,20}$

$$
\begin{equation*}
\mathrm{v}_{0}^{2}=\mathrm{G}\left(\mathrm{M}_{0}+\mathrm{m}_{0 \mathrm{P}}\right)\left(\frac{2}{\mathrm{r}_{0}}-\frac{1}{\mathrm{a}}\right) ; \tag{20}
\end{equation*}
$$

here $\mathrm{m}_{0 \mathrm{P}}$ is the classical mass of the planet and a, the semi-grand axis of the elliptical orbit of this; $a=57.9 \times 10^{6} \mathrm{~km}$; for the case of Mercury.

Throughout our approach, we have assumed the sun infinitely big as compared to Mercury, this being the reason for which the mass of the latter does not appear in our relationships. Below, we shall continue to set all our relationships, that way.

It is further interesting to note that Eq.(20) is nothing but the "classical energy conservation equation"; thus it states that, on the orbit (classically), the total energy of the planet, is conserved.

Classically, the magnitude of the total energy is the energy one has to spend in order to remove the planet bearing a velocity $\mathrm{v}_{0}$, on the orbit at a distance $\mathrm{r}_{0}$ to the sun, from its actual position, to infinity. It is composed of, on the one hand the potential energy, of magnitude $\mathrm{GM}_{0} \mathrm{~m}_{0 \mathrm{P}} / \mathrm{r}_{0}$ (which is the energy one has to spend in order to remove the planet of mass $\mathrm{m}_{0 \mathrm{P}}$, at rest, from a distance $\mathrm{r}_{0}$ to the sun, to infinity), and on the other hand the kinetic energy $(1 / 2) \mathrm{m}_{0 \mathrm{P}} \mathrm{v}_{0}^{2}$.

Thus Eq.(20) states that the magnitude of the classical total energy, i.e. the sum of $\mathrm{GM}_{0} \mathrm{~m}_{0 \mathrm{P}} / \mathrm{r}_{0}$ and $(-1 / 2) \mathrm{m}_{0 \mathrm{P}} \mathrm{v}_{0}^{2}$, on the orbit, must be constant and equal to $\mathrm{GM}_{0} \mathrm{~m}_{0 \mathrm{P}} / 2 \mathrm{a}$.

Having started with Eq.(13), the "relativistic energy conservation equation", it should be natural, as well as fulfilling to land at the "classical energy conservation equation", for small velocities and weak gravitational fields.

Thus for Mercury, D (considering the assumptions in question), shall be given by

$$
\begin{equation*}
\mathrm{D}=1-\frac{\overline{\alpha_{0}}}{2}=1-\frac{\mathrm{GM}_{0}}{2 \mathrm{c}_{0}^{2} \mathrm{a}} . \tag{21}
\end{equation*}
$$

Note that, because $\alpha_{0}$ is small, $D$ is very close to unity. Though the divergence, as small as $\sim 10^{-8}$ from unity, is still essential.

At any rate, following Eq.(13) (giving that the RHS of this equation, is constant), we expect the derivative of $\mathrm{m}_{0}\left(\mathrm{r}_{0}\right)$, or similarly that of $\mathrm{m}_{0}\left(\mathrm{t}_{0}\right)$, with respect to respectively $\mathrm{t}_{0}$ or $\mathrm{r}_{0}$, vanish.

Thence, from Eq.(13), we arrive at the rigorous equation, regarding the revolution of the planet around the sun, or anything as such: ${ }^{8}$

$$
\begin{equation*}
-\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0}^{2}}\left(1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}\right)=\mathrm{v}_{0} \frac{\mathrm{dv}_{0}}{\mathrm{dr}_{0}} . \tag{22}
\end{equation*}
$$

(written by the author, in the local frame of reference)
This relationship is interesting in many ways. First of all when $\mathrm{v}_{0}$ (as compared to the velocity of light) is negligible, or similarly when $\alpha_{0}$ is small, it reduces right away to the classical Newton's equation of gravitational motion. This can be checked immediately by differentiating Eq.(20), which is a scalar form of Newton's equation of gravitational motion.

[^3]As we shall see in Part II, Eq.(22) can further account for the precession of the perihelion of a planet (or anything such), as well as the deflection of light nearby a celestial body, though it is derived through a totally different approach than that of Einstein.

## 6. THE GENERAL EQUATION OF GRAVITATIONAL MOTION IN VECTOR FORM

From a rigorous mathematical point of view, one may argue about the following.

- We does indeed land, from Newton's equation of gravitational motion written in vectorial form, to Eq.(20), ${ }^{19,20}$ thus also to Eq.(22), in the case the cruise velocity $\mathrm{v}_{0}$ of the object in hand is small as compared to the velocity of light. But can we really obtain from the scalar Eq.(22), a corresponding equation in vector form, similar to Newton's (vectorial) equation of gravitational motion?

The answer is

- Yes.

After all we may right away note that our derivation is similar to that of obtaining Newton's equation of gravitational motion, through the classical energy conservation assumption, i.e. the classical Hamiltonian way; ${ }^{19}$ through such an approach the differentiation of Eq.(20) (where we only have to know that the classical total energy amounts to a "constant", clearly without having to know the value of it), yields a scalar differential equation, but which can be converted, as we shall soon swiftly derive, into Newton's equation of gravitational motion in the usual vectorial form.

The energy conservation, in the broader sense of the concept, covering the equivalence of energy of mass, then well, and just similarly, leads to our Eq.(22) as well as the corresponding equation in vectorial form.

Let us elaborate on this. Thus consider the general case, where the magnitude $\mathrm{v}_{0}$, of the velocity vector $\underline{v}_{0}(t)$, changes continuously, all along the motion in question.

In any case, through the infinitely small period of time $\mathrm{dt}_{0}$, we have as usual

$$
\begin{equation*}
\mathrm{d} \underline{\mathrm{v}}_{0}=\underline{\mathrm{v}}_{0}\left(\mathrm{t}_{0}+\mathrm{dt}{ }_{0}\right)-\underline{\mathrm{v}}_{0}\left(\mathrm{t}_{0}\right) . \tag{23}
\end{equation*}
$$

Obviously $\underline{v}_{0}(t)$ and $\underline{\mathrm{v}}_{0}$ are not oriented in the same direction; $\underline{v}_{0}(\mathrm{t})$ is oriented along the direction of the motion on the orbit, whereas $\mathrm{d}_{0}$ is directed toward the sun.

The infinitesimal increase $\mathrm{dv}_{0}$ in the "magnitude" of $\underline{v}_{0}(\mathrm{t})$, i.e.

$$
\begin{equation*}
\mathrm{dv}_{0}=\mathrm{v}_{0}\left(\mathrm{t}_{0}+\mathrm{dt}_{0}\right)-\mathrm{v}_{0}\left(\mathrm{t}_{0}\right), \tag{24}
\end{equation*}
$$

is generally different from $\left|\mathrm{d}_{0}\right|$, the "magnitude of the infinitesimal increase" in $\underline{\mathrm{v}}_{0}(\mathrm{t})$, though $\left|\mathrm{dv}_{0}\right|$ and $\mathrm{dv}_{0}$ become equal, if the motion were a one dimensional motion.

Note that $\mathrm{dv}_{0}$ vanishes in the case of a circular orbit. Recall however that our original equation, i.e. Eq.(13), in this case becomes trivial; thence the differentiation of it, does not provide us with any additional information.

According to the definitions we have made along Eqs. (23) and (24), one can thus show that, ${ }^{19,20}$ the classical Newton's equation of gravitational motion, i.e.

$$
\begin{equation*}
\frac{\mathrm{GM}}{\mathrm{r}_{0}^{2}} \frac{\underline{\mathrm{r}}_{0}}{\mathrm{r}_{0}}=\frac{\mathrm{d} \underline{\mathrm{v}}_{0}}{\mathrm{dt}_{0}} \tag{25}
\end{equation*}
$$

(the classical Newton's equation of gravitational motion, in vector form)
yields well

$$
\begin{align*}
& -\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0}^{2}}=\mathrm{v}_{0} \frac{\mathrm{dv}_{0}}{\mathrm{dr}_{0}} \text {, }  \tag{26}\\
& \text { (the classical Newton's equation } \\
& \text { of gravitational motion, in scalar form) }
\end{align*}
$$

and vice-versa.
Here is a quick proof of our "vice-versa" statement.
Eq.(26) can be classically written as

$$
\begin{equation*}
-\frac{\mathrm{GM}_{0} \mathrm{~m}_{0}}{\mathrm{r}_{0}^{2}} \mathrm{dr}_{0}=-\mathrm{F}_{0 \mathrm{G}} \mathrm{dr}_{0}=\mathrm{m}_{0} \mathrm{v}_{0} \mathrm{dv}_{0} ; \tag{27}
\end{equation*}
$$

this equation expresses that the change in the potential energy and the change in the corresponding kinetic energy are being exchanged.

Recall that here $\mathrm{m}_{0}$ is the classical mass of the planet, and $\mathrm{F}_{\mathrm{OG}}$ the magnitude of the gravitational force between the sun and the planet.

But evidently

$$
\begin{equation*}
\mathrm{F}_{0 \mathrm{G}} \mathrm{dr}_{0}=-\underline{\mathrm{F}}_{0 \mathrm{G}} \cdot \underline{\mathrm{dr}}_{0}, \tag{28}
\end{equation*}
$$

given that the gravitational binding energy is path independent.
Here $\underline{\mathrm{F}}_{0 \mathrm{G}}$ is the gravitational force (in vector form); $\underline{\mathrm{r}}_{0}$ is the location vector defined along $\mathrm{r}_{0} ;\left|\underline{\underline{r}}_{0}\right|$ and $\mathrm{r}_{0}$ are the same quantities;

$$
\begin{align*}
& \mathrm{dr}_{0} \equiv \mathrm{dr}_{0}\left(\mathrm{t}_{0}\right)=\mathrm{r}_{0}\left(\mathrm{t}_{0}+\mathrm{dt}_{0}\right)-\mathrm{r}_{0}\left(\mathrm{t}_{0}\right),  \tag{29}\\
& \mathrm{dr} \underline{\mathrm{r}}_{0} \equiv \mathrm{dr}_{0}\left(\mathrm{t}_{0}\right)=\underline{\mathrm{r}}_{0}\left(\mathrm{t}_{0}+\mathrm{dt}_{0}\right)-\underline{\mathrm{r}}_{0}\left(\mathrm{t}_{0}\right) . \tag{30}
\end{align*}
$$

The negative sign at the RHS of Eq.(28) arises from the fact that, as $\mathrm{r}_{0}$ increases, the force counteracts, making the cosine of the dot product negative (or the same, as $\mathrm{r}_{0}$ decreases, the force acts in speeding up the motion, making the cosine of the dot product, positive).

We now rewrite Eq.(27), dividing its both sides by $\mathrm{dt}_{0}$ :

$$
\begin{equation*}
\underline{\mathrm{F}}_{0 \mathrm{G}} \cdot \underline{\mathrm{v}}_{0}=\mathrm{m}_{0 \mathrm{P}} \mathrm{v}_{0} \frac{\mathrm{dv}_{0}}{\mathrm{dt}_{0}} \tag{31}
\end{equation*}
$$

where we made use of the usual definition of $\underline{v}_{0}$, i.e.

$$
\begin{equation*}
\underline{\mathrm{v}}_{0}=\frac{\mathrm{dr}_{0}}{\mathrm{dt}_{0}} . \tag{32}
\end{equation*}
$$

Let us multiply both sides of Eq.(31) by d $\underline{v}_{0}$ [cf. Eq.(23)], and rearrange it:

$$
\begin{equation*}
\left|\underline{\mathrm{F}}_{0 \mathrm{G}}\right| \frac{\left|\underline{\mathrm{v}}_{0}\right|}{\mathrm{v}_{0}} \cos \theta \frac{\left|\underline{\mathrm{~d}}_{0}\right|}{\mathrm{dv}_{0}}=\mathrm{m}_{0 \mathrm{P}} \frac{\left|\underline{\mathrm{~d}}_{0}\right|}{\mathrm{dt}_{0}} \tag{33}
\end{equation*}
$$

here $\theta$ is the angle between $\underline{\mathrm{F}}_{0 \mathrm{G}}$ (directed toward the sun), and $\underline{\mathrm{v}}_{0}$ (tangent to the orbit). [Bear in mind that $\mathrm{v}_{0}$ is identical to $\left|\mathrm{v}_{0}\right|$.]

One can on the other hand, easily show that

$$
\begin{equation*}
\mathrm{dv}_{0}=\left|\mathrm{d}_{0}\right| \cos \theta, \tag{34}
\end{equation*}
$$

checking at once the case of the circular motion, for which $\theta=\pi / 2$, and $\mathrm{dv}_{0}=0$; one can moreover note that this also checks well the sign of $\mathrm{dv}_{0} .{ }^{* *}$

Furthermore $\mathrm{dv}_{0}$ is directed toward the sun, along the same direction as $\underline{\mathrm{F}}_{0 \mathrm{G}}$. This makes that Eq.(26) written in scalar form, yields well Eq.(25) written in vectorial form (c.q.f.d.).

Based on the foregoing information, it becomes clear that starting with our Eq.(22), we can obtain the vectorial equation

$$
\begin{equation*}
\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0}^{2}}\left(1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}\right) \frac{\underline{\mathrm{r}}_{0}}{\mathrm{r}_{0}}=\frac{\mathrm{d} \underline{\mathrm{v}}_{0}}{\mathrm{dt}_{0}} \tag{35}
\end{equation*}
$$

> (the general equation of gravitational motion written by the author, in the local frame of reference)

[^4]Unless $\mathrm{v}_{0}$ is small, this relationship displays an amazing feature; it is that the "gravitational mass" and the "inertial mass" are not the same. We shall elaborate on this in what follows.

Regarding the motion of a planet around the sun, the classical energy conservation, via the Hamiltonian approach yields well Newton's second law of motion, i.e.

$$
\text { Force }=m_{0 \mathrm{p}} \times \text { Acceleration, }
$$

or the same,

$$
\text { Gravitational Field }(\text { Vector })=\text { Acceleration }(\text { Vector }) .
$$

The approach we presented herein, via the relativistic energy conservation, clearly, does not yield Newton's second law of motion; it yields something else.

In order to draw a one to one comparison between the frame we just sketched [through Eqs. (25) - (35)], and our approach, we would like to rewrite our Eq.(22), out of our Eq.(13), and reexamine it:

$$
\begin{equation*}
-\frac{\mathrm{GM}_{0}}{\mathrm{c}_{0}^{2} \mathrm{r}_{0}^{2}} \frac{\mathrm{~m}_{0 \infty} \mathrm{e}^{-\alpha_{0}}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}} \mathrm{dr}_{0}=-\frac{-2 \frac{\mathrm{v}_{0} \mathrm{dv}_{0}}{\mathrm{~m}_{0 \infty} \mathrm{e}^{-\alpha_{0}}} \frac{2 \sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}}{\left(1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}\right)}}{} . \tag{36}
\end{equation*}
$$

[Eq.(22), rewritten by differentiating Eq.(13)]
The LHS of this equation expresses the infinitesimal change in the gravitational binding energy of the object in motion.

The RHS conversely expresses the infinitesimal change in the kinetic energy of the "overall mass" $\mathrm{m}_{0 \mathrm{e}} \mathrm{e}^{-\alpha_{0}} / \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$; recall that this mass remains as a constant throughout [cf. Eq.(13)]. Note that the change on the kinetic energy, is solely due to the change on the velocity.

Thence by rereading Eq.(36), along the derivation of Newton equation of gravitational motion [Eq.(26)] we can state that

$$
\begin{equation*}
\text { Gravitational Force }=\frac{(\text { Overall Mass })(\text { Acceleration })}{\left(1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}\right)} ; \tag{37}
\end{equation*}
$$

(the general equation of gravitational motion written by the author)
here the gravitational force, next to the sun's mass (assumed at rest), embodies the overall mass, $\mathrm{m}_{0 \mathrm{x}} \mathrm{e}^{-\alpha_{0}} / \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$ of the revolving object.

Eq.(37), reduces to Eq.(22), once we divide both of its sides by the overall mass.
Eq.(37), based on the analysis we made on Eq.(36), seems the natural way of presenting our result. Accordingly one uses the same mass, i.e. $\mathrm{m}_{0 \mathrm{ox}} \mathrm{e}^{-\alpha_{0}} / \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$, to multiply both the gravitational field intensity and the acceleration. But then Newton's equation of gravitational motion, i.e. [Force $=\mathrm{m}_{0 \mathrm{P}} \times$ Acceleration] is broken.

Formally, this can be saved if instead, we choose to alter the "gravitational force" term; but then the gravitational mass and the inertial mass, as classically defined, shall not be same.
We conclude on this below.

## 7. CONCLUSION

The essence of this article was, based the energy conservation, in the broader sense of the concept, embodying the equivalence of mass and energy, to derive a general equation of gravitational motion, more specifically

$$
\begin{equation*}
\frac{\mathrm{GM}_{0}}{\mathrm{r}_{0}^{2}}\left(1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}\right) \frac{\underline{\mathrm{r}}_{0}}{\mathrm{r}_{0}}=\frac{\mathrm{dv}_{0}}{\mathrm{dt}_{0}} \tag{35}
\end{equation*}
$$

(the general equation of gravitational motion written by the author, in the local frame of reference)

This becomes the Newton's equation of motion, only if $\mathrm{v}_{0}$ is small as compared to the velocity of light. In Part II, we shall see, how this equation can cover up the basic predictions of the general theory of relativity, provided that we take into consideration the fact that the mass deficiency due to the binding, alters via quantum mechanics, the unit lengths, unit periods of time, etc.

The way it stands though, the principle of equivalence of the gravitational mass and the inertial mass, in general, seems in trouble.

This principle is anyway severely questioned. ${ }^{21,22,23}$
Nonetheless we can formally save Newton's equation of gravitational motion, by redefining the gravitational mass.

Thus consider the classical formulation of Newton's equation of gravitational motion, tuned along the special theory of relativity, i.e. with the familiar notation ${ }^{11}$

> Classically expressed Gravitational Force $=\left[\mathrm{d}(\right.$ Momentum of the object in motion, due to gravitation $\left.) / \mathrm{dt}_{0}\right]$.

To ease our expression, let us continue to consider, say Mercury of mass $\mathrm{m}_{0 \infty}$, at infinity, in its motion around the sun of mass $M_{0}$, without however any loss of generality.

Note that here we assume, we are positioned at Mercury. Things will be seen differently, when we will be positioned at a distance far away from the sun's gravitational field. This latter situation shall be undertaken in Part II.

Comparing Eqs. (35) and (36), the mass $\mathrm{m}_{01}$, pertaining to the planet, and entering the formulation of the momentum of the planet, shall be $\mathrm{m}_{0 \mathrm{x}} \mathrm{e}^{-\alpha_{0}} / \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$; this corresponds to the classical inertial mass; it is a constant of our approach, therefore it comes out of the "differentiation operation" on the momentum.

Let us then call $\mathrm{m}_{0 \mathrm{G}}$, a gravitational mass pertaining to the planet, taking part in the usual gravitational force acting between the sun and the planet, so that

$$
\begin{equation*}
\mathrm{G} \frac{\mathrm{M}_{0} \mathrm{~m}_{0 \mathrm{G}}}{\mathrm{r}_{0}^{2}} \frac{\mathrm{r}_{0}}{\mathrm{r}_{0}}=\mathrm{m}_{01} \frac{\mathrm{~d} \underline{\mathrm{v}}_{0}}{\mathrm{dt}_{0}}=\frac{\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}}}{\sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}}} \frac{\mathrm{~d} \underline{\mathrm{v}}_{0}}{\mathrm{dt}_{0}} . \tag{39}
\end{equation*}
$$

[Eq.(38) expressed via the use of the gravitational mass]
This latter equation becomes the same as Eq.(35), if we propose to write

$$
\begin{equation*}
\mathrm{m}_{0 \mathrm{G}}=\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}} \sqrt{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}} . \tag{40}
\end{equation*}
$$

## (gravitational mass that would take part in the classical gravitational force expression, as assessed by the local observer)

Our result, at any rate leads us to a straightforward conclusion, albeit totally against the prevailing wisdom; it is worth to state it as a separate theorem.

Theorem 4: The gravitational mass $\mathrm{m}_{0 \mathrm{G}}$, and the inertial mass $\mathrm{m}_{0 \mathrm{I}}$, as classically defined, are not the same; the theory presented herein, to formally save Newton's equation of gravitational motion, predicts $\mathrm{m}_{0 \mathrm{G}}=\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}} \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$, given that $\mathrm{m}_{0 \mathrm{I}}=\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha_{0}} / \sqrt{1-\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}}$; though undetectable, for most cases we observe, $\mathrm{m}_{0 \mathrm{G}}$ and $\mathrm{m}_{01}$ differ.

The equality of the gravitational mass and the inertial mass, based on our approach is an approximation which is acceptable, only if the velocity of the object in motion is small. Thus the equality between the gravitational mass of the object in hand, and its inertial mass, is valid if there is no motion, or the same, when the observer is embarked in the motion.

It is indeed interesting to note that all the highly precise measurements regarding the relative divergence of these two masses, are performed on Earth (where the observer is moving with Earth), so that the precision they produce, no matter how fine this may be, should be considered, as misleading.

Given that the gravitational mass, as stated by Eq.(40), depends on the velocity, one should not rely on the experiments in question, any more then he should count on the null result of the Michelson Morley experiment ${ }^{24}$ (which, being performed on Earth, fails to detect the motion of Earth around the sun, or else). In fact, it should be recalled that the principle of relativity (the main ingredient of the special theory of relativity), forbids that we can on Earth, detect any such difference, based on the velocity of motion in question, since otherwise we should be able to tell accordingly, how fast we are cruising in space.

One may still insist (just the way it is done regarding the experiments in question) that he can measure the difference between the gravitational mass and the inertial mass, on Earth. But the rotational velocity $\mathrm{v}_{0}$ of Earth around itself is $1667 \mathrm{~km} / \mathrm{hour}$. Hence one should attain a precision of $\mathrm{v}_{0}^{2} / \mathrm{c}_{0}^{2}$, i.e. better than $2.6 \times 10^{-12}$, whereas the highest precision reached so far, is bearly, this much.

On the other hand, measurements based on a possible polarization of Earth and the Moon, through their motion around the sun, (on which we can indeed rely), require a precision of $\sim 10^{-8}$ (which is the related ratio of $\mathrm{v}_{0}^{2}$ to $\mathrm{c}_{0}^{2}$ ), whereas the precisions actually reached $\left(\sim 10^{-4}\right)$, happen to be far below this. ${ }^{25,26}$

In contrast it is astoundingly interesting to note that Eq.(22) can be obtained from the following equation bearing the same form as that of the classical Newton Equation of Motion, i.e. Eq.(26):

$$
\begin{equation*}
-\frac{G M_{0}\left(\mathrm{~m}_{0 \infty} \mathrm{c}^{\alpha_{0}}\right)}{\mathrm{r}_{0}^{2}}=\mathrm{v}_{0} \frac{\mathrm{~d}\left(\mathrm{~m}_{0 \infty} \mathrm{\alpha}^{\alpha_{0}} \mathrm{v}_{0}\right)}{\mathrm{dr}_{0}} \tag{41}
\end{equation*}
$$

This means that, if the local mass $\mathrm{m}_{\mathrm{oL}}$ were given by,

$$
\begin{equation*}
\mathrm{m}_{0 \mathrm{~L}}=\mathrm{m}_{0 \infty} \mathrm{e}^{\alpha_{0}}, \tag{42}
\end{equation*}
$$

instead of that given by Eq.(11) (i.e. $\mathrm{m}_{0 \mathrm{~L}}=\mathrm{m}_{0 \infty} \mathrm{e}^{-\alpha}$ ), and if the local relativistic effect due to the translational motion of the object of concern can be ignored [since the momentum quantity, expressed as $m_{0 \infty} \mathrm{e}^{\alpha_{0}} \mathrm{v}_{0}$ under the differentiation operation at the RHS of Eq.(17) does not cover the effect due to the translational motion of the object], only then we could claim that the principle of equivalence holds, i.e. the gravitational mass and the inertial mass are the same.

But this is not the case; that is, through our approach, Eq.(42) is incorrect; furthermore the local relativistic effect due to the translational motion of the object, in general, cannot be ignored. Thus the principle of equivalence must be incorrect.

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[^0]:    * The Rydberg Constant $\mathrm{RC}_{0}$, were it measured in empty space, is

    $$
    \mathrm{RC} C_{0}=\frac{2 \pi^{2} \mathrm{~m}_{\mathrm{e} 0} \mathrm{e}^{4}}{\mathrm{c}_{0} \mathrm{~h}^{3}}
    $$

    here, $\mathrm{m}_{\mathrm{e} 0}$ is the mass of the electron in empty space; e is the electron charge intensity, assumed to stay unaltered in the gravitational field; the Planck Constant h , too, according to our theory remains unaltered in the gravitational field. Thus at the RHS of the above relationship, we expect only $\mathrm{m}_{\mathrm{e} 0}$, to get altered in the gravitational field. On the other hand $\mathrm{c}_{0}$, the speed of light in empty space, is not locally altered; anyway, the introduction of it in the above expression, is a matter of expressing the Rydberg Constant in $\mathrm{cm}^{-1}$ unit; otherwise it would bear an "energy", more precisely a "frequency" dimension.

[^1]:    $\dagger$ The gravitational potential $\mathrm{V}(\mathrm{r})$, in the vicinity of a celestial body of mass $M_{0}$, furnished by the general theory of relativity is ${ }^{6}$

[^2]:    * Based on Eq.(13), and the data, for instance,

    $$
    \begin{aligned}
    \mathrm{r}_{\text {Operihilion }} & =46.0 \times 10^{6} \mathrm{~km}, \quad \mathrm{v}_{\text {Operibilion }}=58.98 \times \mathrm{km} / \mathrm{s}, \\
    \mathrm{r}_{\text {Oaphelion }} & =69.8 \times 10^{6} \mathrm{~km}, \quad \mathrm{v}_{\text {Oaphelion }}=38.86 \times \mathrm{km} / \mathrm{s}
    \end{aligned}
    $$

[^3]:    ${ }^{8}$ In the case we consider the electron revolving on an elliptic orbit around the nucleus, this equation [via Eqs. (3), (4) (5), (12) and (13), this time, written for the electron bound to the nucleus], in CGS unit system, becomes

    $$
    -\frac{\mathrm{Ze}^{2}}{\mathrm{~m}_{\mathrm{ec}} \mathrm{c}_{0}^{2} \mathrm{r}_{0}^{2}} \frac{1}{1-\frac{\mathrm{Ze}^{2}}{\mathrm{~m}_{\mathrm{e} \mathrm{r}_{0} \mathrm{c}_{0}^{2}}^{2}}}=\frac{1}{1-\frac{\mathrm{v}_{0}^{2}}{\mathrm{c}_{0}^{2}}} \frac{\mathrm{v}_{0}}{\mathrm{c}_{0}^{2}} \frac{\mathrm{dv}}{\mathrm{dr}_{0}} \text {; }
    $$

    (written by the author, in the local frame of reference for the electron revolving around the nucleus)
    here as usual, $\mathrm{m}_{\mathrm{e}}$ is the electron mass at infinity, e the charge of the electron, and Ze the charge of the nucleus. (According to our approach, this is the correct equation which should have been written by Sommerfeld.)

[^4]:    ${ }^{* *}$ Note that, just likewise, one can write

    $$
    \mathrm{dr}_{0}=-\left|\mathrm{dr}_{0}\right| \cos \theta \text {, }
    $$

    instead of Eq.(28), and accordingly the path independency of the gravitational binding energy can well be proven.

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