MASS DEFICIENCY CORRECTION TO THE RELATIVISTIC QUANTUM MECHANICAL APPROACH: METRIC CHANGE NEARBY THE NUCLEUS

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ABSTRACT

The mass of an electron bound to a nucleus, due to the "equivalence of mass and energy", based on the special theory of relativity, should decrease, as much as the binding energy this electron delineates. This, so far, seems to have been overlooked. The magnitude of the bound electron's mass decrease, is an effect about twice the magnitude of the Dirac's relativistic effect. Furthermore, it is in the opposite direction, which makes that the energy levels, contrary to what Dirac had predicted, should be (as compared to Bohr's, as well as Sommerfeld's energy levels) shifted (not downward, but), upward. The magnitude of this shift, surprizingly, turns out to be just as much that of Dirac's shift, i.e. $1.81x10^{-4}$ ev for n=1, and $1.15\ x10^{-5}$ ev for n=2, for the H atom (and upward); the shift for (l=1, n=2) is about %10 of the shift for (l=1, n=2). Thus the bound electron's mass decrease, may partly eradicate some of the diverse speculations designed to cover the "upward shift anomalies", Dirac's theory does not predict, such as Lamb effect, estabished more than half a century ago. The mass decrease of the bound electron, on the other hand, amazingly induces at once, a "change of the metric nearby the nucleus". The cast of our approach furthermore (contrary to Sommerfeld's framework), remains applicable to the precession of the perihilion of the planets around the sun.

1. INTRODUCTION: OWING TO THE EQUIVALENCE BETWEEN MASS AND ENERGY, THE *PROPER MASS* CAN WELL BE ALTERED.

Dirac, composed his *relativistic wave equation,* together with an *elegant solution,* by taking into account the *relativistic behaviour* of the electron(s) around the nucleus.[1] Sommerfeld, [2] had already improved previously, Bohr's Atom Model,[3] based on the same idea.

Neither of them however, had considered the fact that the *mass of the electron,* as well as *(though in a slim proportion)* that of the proton, owing to the *equivalence between mass and energy,* should decrease when bound to each other, still as implied by the special theory of relativity.[4, 5]

Apparently, noone afterwards had registered a thought regarding the fact that the *proper mass (i.e. the rest mass in empty space)* of the electron, just like *that of the proton (in general, that of the nucleus)* should decrease, when these form a bond with each other.

The reason such a process has been overlooked, is presumably the fact that *as a first strike*, it looks indeed conflicting that, *an elementary particle would in anyway be altered, through an "ordinary" electrostatic interaction with an other one*.

However, we know that the proton and the electron, when bound to each other in the hydrogen atom, weigh less than the sum of the proton and the electron, carried away from each other; the mass deficiency in question is (by taking the speed of light, unity), exactly equal to the binding energy of the proton and the electron in the hydrogen atom, i.e. 13.6 ev, based on the fundamental relationship, about the relativistic equivalence between mass and energy 5

(Energy released, or acquired) = (Magnitude of the algebraic increase in the mass)
x (Speed of light in empty space)^2 .

So, contrary to the widespread opinion, the *electron* or the *proton* cannot be the *same*, when bound to each other; they are different. Their internal dynamics altogether, thus *weaken* as much as 13.6 ev, when they are bound to each other, to shape up the *hydrogen atom.*

Many scientists though, still *firmly* think that there is the *"proper mass" (rest mass)* and the *"relativistic mass" (defined within the frame of the special theory of relativity),* and that the *proper mass* is, whatsoever an *invariant,* which is a *characteristic of matter,* and that is all. Generally speaking, this is unacceptable. The *proper mass* of a given particle on the whole, *at rest* may, depending on the circumstances, embody a *more or less energetic internal motion;* this will, *one way or the other,* affect the *proper mass.*

Suppose indeed that Captain Electron *(we mean, the electron itself)* is cruising in a *full electric isolation,* with a *uniform translational velocity.* So does Captain Proton *(i.e. the proton itself).* They approach to each other. Then *(based on the special theory of relativity)* we would be certain that, Captain Electron in its own frame of reference, all the way through, preserves its identity, defined at infinity. *(So will also do Captain Proton.)* If now, we remove the *previous electric isolation,* Captain Electron and Captain Proton, because of the *electric attraction force,* they mutually create, shall start getting accelerated toward each other. The *"extra kinetic energy"* they would acquire through this process, shall be supplied by the *system made of the two.* Their total energy [i.e. (the sum of their relativistic masses) x (the speed of light)^2], through the motion, shall remain *constant,* and *equal* to the equivalent of the sum of their initial relativistic masses. *(Otherwise, the energy conservation law would be broken.)* Let us suppose for simplicity that in the latter case *(where we have no electric isolation),* they start, far away from each other, *at rest;* then their initial relativistic masses are, essentially identical to respectively their *rest masses.* If now the accelerating *Captain Electron,* say in *Captain Proton's frame of reference,* hurts an *obstacle* and looses all the kinetic energy, it would have acquired through the attraction process; thence, it must *concurrently* dump a *portion* of its *rest mass,* and this, as much as the amount of the kinetic energy it would have piled up, on the way.*

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* It was an incomparable privilege to have discussed with Professor R. Feynman, the very first seed of the idea presented herein, and to have been encouraged with his support, through a Fulbright visiting stay at California Institute of Technology, back in 1984. It is also a privilege to have been backed up by Professor Rozanov, Head of Laser PlasmaTheory Department of Lebedev Institute, Moscow.

Thus, we cannot say that the proton and the electron are the same, after we have retrieved from the system made of the two, a given amount of energy, no matter how much. The greater is the energy extracted, the harder will be the harm caused in their internal dynamics, conseqeuntly in their *proper masses* defined at infinity.

This is exactly what happens when, say the hydrogen atom is formed, except that the electron, as referenced to the proton is not anymore at rest, but possesses a given amount of kinetic energy; an energy of 13.6 ev is needed, to carry the electron away from the proton, back to infinity.

It is thus clear that *as referenced to the proton,* or *(since the proton is much too big as compared to the electron)* practically the same, *as referenced to the "laboratory system",* the *hydrogen electron's proper* (*rest) mass,* is altered as much.

Just the same way, the daily *production of thermal energy,* is due to the transformation of a *minimal part of the mass* entering in reaction, into energy. Thus, the reaction products weigh less than the reactants, and this, as much as the *energy* produced throughout.

The fuel, i.e. *coal, petroleum, uranium, plutonium,* anything, in a power plant of, say 3000 $MW_{thermal}$, continuously working for a period of one year, thus producing an energy amounting to 3000 $MW_{thermal}$ x year, at the end of this period, weighs less, as much as the equivalent of the energy output in question, i.e. [based on the equivalence between mass and energy], about 1 kg. This is of course insignificant as compared to *millions of tons (whatever shall be the approximate amount)* of coal or petroleum that would be fired into the plant of concern, but well detectable as compared to about a *ton* of plutonium-239, or uranium-235 needed to be depleted in a *nuclear power plant* of 3000 $MW_{thermal}$ through a period of one year.

In a similar way, a *compressed spring* should be heavier than the *"same spring"* when stretched out; or the *gas in a room at a high temperature* should weigh more than the *"same gas"* at a lower temperature, etc.

All these, already happen to be well established facts. Thus, any *proper mass* weighs less, after releasing energy, or conversely it shall weigh more, after piling up an *extra amount of internal energy.*

Recall that in the case of a *nuclear fission,* we just referred to, it is the *increase in the binding energy* of the nucleons in the nuclei of the fission products, *in comparison with the binding energy of the nucleons in the original plutonium-239, or uranium-235 nuclei, in question,* which is responsible of the *nuclear energy* released; thence the *nuclear binding energy* has well a *mass deficit counterpart.* Why shouldn't the bound electron? Thus, once again, it seems unconceivable, not to associate with the bound electron, a *mass deficit.*

Below, we *first* review the *inventory* marked out by the bound electron's *over all relativistic mass* (Section 2). The *constancy* of this, at a given energy level, is a *must* imposed by the *energy conservation law,* yielding via *differentiation*, a *novel equation for the electron's motion around the nucleus,* as a simplest framework *(which we call the Modified Bohr Atom Model),* we can set up, following our approach (Section 3). This provides us, with the possibility of getting a quick, but exact expression regarding the *new energy levels (as far as the principal quantum numbers they involve, are concerned)* (Sections 4 and 5). The idea of the *change of the metric neraby the nucleus,* immediately follows up as a main conclusion of our approach (Section 6).

2. THE *OVERALL MASS* OF THE ELECTRON IN THE ATOM ON A GIVEN LOCATION, IS *ITS PROPER MASS DECREASED AS MUCH AS ITS BINDING ENERGY AT REST, AT THIS LOCATION, AND INCREASED BY THE LORENTZ FACTOR, DUE TO ITS MOTION*

Based on the foregoing discussion, henceforth, we should take into account the *proper mass decrease of the bound electron,* as implied by the *special theroy of relativity.*

More specifically we can think that, the hydrogen atom is made in two steps:

1) We bring the electron from infinity to a given distance from the nucleus *(supposing for simplicity, yet without any loss of generality, that the proton is fixed);* owing to the *equivalence between mass and energy,* this process reduces the electron's *proper mass* as much as its *potential energy* at this location.

2) Next, we deliver to the electron its orbital kinetic energy *(however, the orbit, may be conceived);* this process, in the familiar relativistic way, increases the already decreased *proper mass,* by the usual Lorentz factor.

Thus Dirac, just like Sommerfeld had considered the *second process,* but not the *first one.*

It is in effect known that, Dirac's predictions do not cover thoroughly the experimental results.[6,7,8,9] The doublets due to *spin-orbit interaction* are indeed somewhat narrower than predicted. There is also, though very little, a *shift* of energy levels, *upward (whereas the "relativistic quantum mechanics", just like "Sommerfeld's approach", predicts a shift of the Bohr energy, downward).*

Theoretical explanations provided for these anomalies, such as a *perturbing repulsive interaction between the electron and the nucleus,* do not seem to cover thoroughly the reality.

In this communication we are going to show that, the anomalies in question are mostly due to the fact that, Dirac's theory did not take into account the *proper mass decrease of the bound electron.* The *order of magnitude* of the correction that we are going to calculate based on this, turns out to be just as much as the observed deviation from the classical Dirac theory.

We are going to base our approach on just the *hydrogenlike atoms.* Further for *simplicity (though without any loss of generality),* we shall neglect the *mass deficiency* undergone by the proton in the hydrogen atom, as compared to that displayed by the electron; along the same line, we can consider that, the *reduced mass* of the electron and the proton, is the mass of the electron, *straight.*

We thus make the following definitions.

r_0	:	distance of the electron to the nucleus,
$m_{0\infty}$	:	the electron's rest mass at infinity
$m_0(r_0)$	:	the electron's rest mass at a distance r_0 from the nucleus
$m(r_0)$	:	the electron's overall mass *(which is its mass at infinity, decreased as much as its potential energy, and increased based on the special theory of relativity, due to its "translational" motion)* at r_0
v_0	:	the tangential velocity of the electron on the *orbit (however the motion or the orbit may be conceived),* at the location r_0
c_0	:	the velocity of light in empty space
e	:	the charge intensity of the electron or that of the proton
Z	:	the number of protons of the nucleus

Thence the *overall mass* $m(r_0)$ of the electron, at a distance r_0 from the nucleus bearing Z protons, can be written as

$$m(r_0) = m_0(r_0)\frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = m_{0\infty}\frac{1-\frac{Ze^2}{m_{0\infty}r_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \text{Consant on an Orbit} \quad ; \qquad (1)$$

(overall mass of the electron, in a hydrogenlike atom, written by the author)

here, $1-Ze^2/(m_{0\infty}r_0c_0^2)$ is the *decrease factor* of the *proper mass* $m_{0\infty}$ of the electron, when bound, *at rest,* to the nucleus of concern; thus Ze^2/r_0 as usual, is the *potential energy,* or the same, the *binding energy at rest* of the electron to this nucleus, at a distance r_0 from it, which makes that $Ze^2/(m_{0\infty}r_0c_0^2)$ is the ratio of the *potential energy* to the *original proper energy.*

According to our approach, it is in fact the *decreased mass at rest,* $m_0(r_0)$ at r_0, which is increased by the *Lorentz factor* $1/\sqrt{1-v_0^2/c_0^2}$, due to the electron's motion around the nucleus *(and not the proper mass* $m_{0\infty}$*, measured in empty space, free of any field).*

Thus Dirac's theory, just like Sommerfeld's approach, misses the *decrease factor* $1-Ze^2/(m_{0\infty}r_0c_0^2)$ we introduced in Eq.(1). This factor, as will soon become clear, is very small for small Z's, but may become quite important at big Z's; anyway *(as we shall elaborate below)* the *inverse* of it is amazingly equal to the *square of the Lorentz factor* meaning that the overall mass *(contrary to the actual wisdom and related mathematical formulation)*, is always smaller than $m_{0\infty}$.

Note that setting the RHS of Eq.(1) equal to a *constant*, determines the *orbit equation* of the electron around the nucleus, given that *on the orbit*, whether this is a circle, or an ellipse, or else *(anyway we can conceive)*, *the total energy*, i.e. $m(r_0)c_0^2$, ought to be *constant.*

One way of quickly assessing the effect of taking into account *the mass decrease of the proper mass of the electron*, is to use the Bohr Atom Model, straight. Thus the ratio $Ze^2/(m_{0\infty}r_0c_0^2)$ can, *as a first approach* be evaluated, using the familiar relationship

$$4\pi^2 Ze^2 m_{0\infty} r_0 = n^2 h^2 \ , \qquad (2)$$

where n is the Bohr quantum number.

Below, we shall refine our assessment by writing a *Modified Bohr Theory*, chiefly based on Eq.(1).

Via Eq.(2), we can now evaluate the factor $1-Ze^2/(m_{0\infty}r_0c_0^2)$, appearing in Eq.(1), essentially for small Z's:

$$\left(1-\frac{Ze^2}{m_{0\infty}r_0c_0^2}\right)=1-\frac{1}{n^2}Z^2\alpha^2 \ , \qquad (3)$$

where α is the *fine structure coefficient*, i.e.

$$\alpha=\frac{2\pi e^2}{hc_0^2}\cong\frac{1}{137} \ . \qquad (4)$$

On the other hand, the *mass dilation factor* $1/\sqrt{1-v_0^2/c_0^2}$, can also be quickly evaluated, still for small Z's, based on the results of Bohr Atom Model, i.e. Eq.(2), and the expression for v_0, the *velocity of the electron*, at the given energy level denominated by n, or

$$v_0=\frac{2\pi e^2}{nh} , \qquad (5)$$

leading to

$$\frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\cong 1+\frac{1}{2n^2}Z^2\alpha^2 \ . \qquad (6)$$

So, as we see, not only that the *mass decrease factor* and the *Lorentz factor* bear the *same order of magnitude*, but also, the *magnitude of the correction* $Z^2\alpha^2/n^2$, due to the *mass decrease* is already *twice* as that due to the Lorentz factor; moreover the *mass decrease correction* works as to cancel the Lorentz dilation.

Eqs. (3) and (6), for small Z's, yield the *over all mass* $m(r_0)$ at r_0:

$$m(r_0) = m_{0\infty} \frac{1-\frac{Ze^2}{m_{0\infty} r_0 c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \cong m_{0\infty}(1-\frac{1}{2n^2}Z^2\alpha^2)\,. \tag{7}$$

(overall mass of the electron in an hydrogenlike atom, written by the author)

The *difference* between Dirac's overall mass *(the proper mass dilated by the Lorentz factor),* and our's, is then $m_{0\infty}Z^2\alpha^2/n^2$. [Dirac's *overall mass* is larger than $m_{0\infty}$ by $m_{0\infty}Z^2\alpha^2/(2n^2)$; our's is smaller than $m_{0\infty}$ by still $m_{0\infty}Z^2\alpha^2/(2n^2)$.]

At this stage it would be interesting to investigate the *effect of the mass decrease* of the bound electron, based on a *simplest model* we can develop, by modifying Bohr Atom Model, based on Eq.(1).

3. THE INVESTIGATION OF THE EFFECT OF MASS DECREASE OF THE ELECTRON, BASED ON A *MODIFIED BOHR ATOM MODEL*

The *differentiation* of Eq.(1) yields the following noteworthy, *general orbit equation,* for the motion of the electron around the nucleus:

$$-\frac{Ze^2}{m_{0\infty} r_0^2}\frac{1-\frac{v_0^2}{c_0^2}}{1-\frac{Ze^2}{m_{0\infty} r_0 c_0^2}} = v_0\frac{dv_0}{dr_0}\quad. \tag{8}$$

One can transform this equation into a *vector equation,* with not much pain, and show that the RHS, is accordingly transformed into the *acceleration (vector)* of the electron on the orbit.

The orbit would be as customary *elliptical,* for a small Z, thus a small v; otherwise it is open; in other words, the perihelion of it, shall precess throughout the motion.

Note that Eq.(8), is different than that written by Sommerfeld *(due to the fact that, he had not considered the mass decrease of the bound electron).*

It can be shown that the [(-) RHS] of Eq.(8) is equal to the magnitude of the acceleration. At the *limiting case,* for a circular motion of the electron around the proton this quantity shall thus become

$$-v_0 \frac{dv_0}{dr_0} = \frac{v_0^2}{r_0} \quad . \tag{9}$$

We can consequently rewrite Eq.(8), using Eqs. (1) and (9), for a *circular orbit:*

$$\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}} = m(r_0)\frac{v_0^2}{r_0} \quad . \tag{10}$$

(equation written by the author for the electron moving on a circular orbit around the nucleus)

This is anyway the same relationship as that proposed by Bohr, except that the *electrostatic force intensity* is now decreased by the factor $\sqrt{1-v_0^2/c_0^2}$.

Next, we should write the Bohr's postulate in an *appropriate* way. For this, it would be useful to recall de Broglie's doctorate thesis.[10] Along this approach, Bohr's postulate reduces to the expression of de Broglie wave *(associated with the electron's motion),* confined *(thus, like any classical wave, bound to be quantized)* on the orbit. But the *momentum* of the electron entering the de Broglie's relationship, must be the *local relativistic momentum,* where then the *"mass"* should be taken as the *overall mass,* we defined at the stage of Eq.(1).

Thus we propose to write

$$2\pi m(r_0)v_0 r_0 = nh \text{ , n=1, 2, 3, ...} \tag{11}$$

(de Broglie's relationship rewritten by the author, instead of Bohr's postulate, taking into account the overall mass decrease of the bound and confined electron)

We call the set of equations Eqs. (8) and (11) the *Modified Bohr Equations.* The *two unknowns* v_{0n} and r_{0n} *(to be associated with the nth quantum level),* for *circular orbits* can then be found to be

$$v_{0n} = \frac{1}{\sqrt{1+\frac{n^2}{Z^2\alpha^2}}}c_0 \ , \tag{12}$$

and

$$r_{0n} = \frac{Ze^2}{m_{0\infty}c_0^2}\left(1+\frac{n^2}{Z^2\alpha^2}\right) \ . \tag{13}$$

These *two simple relationships* are interesting in several ways.

First of all, for small Z's we land back to Bohr's results. The orbit velocity, v_{0n} as expected, cannot increase beyond the *velocity of light,* no matter how big Z is (cf. Figure 1). As Z increases, the orbit radius r_{0n} decreases to draw a *minimum* at $Z\alpha/n = 1$(cf. Figure 2); the value of the radius r_{0nmin} at the minimum, is $2Ze^2/(m_{0\infty}c_0^2)$, where then, *half of the proper mass of the electron* would disappear; r_{0nmin} for n=1, becomes 0.77 x 10^{-10} cm, i.e. ~ 1/100th of the *Bohr Radius;* the value of the velocity v_{0nmin} at the minimum r_{0nmin} is $c_0/\sqrt{2}$; the *subsequent increase* of r_{0nmin} for big Z's is practically *linear.*

The reason r_{0n} decreases for small Z's, is that *(though the magnitude of the effect of the mass decrease of the bound electron is greater than the magnitude of the effect of the relaticvisitic mass increase),* the *mass change* is anyway negligible; thus the decrease is basically due to the increase of Z; but for big Z's, despite the Z increase, the mass decrease becomes more important [cf. Eq.(7)], which altogether, in harmony with Eq.(2), pulls the orbit outward. Following the *linear increase* for big Z's, r_{0n} *finally* catches up, with is value for Z=1, when Z reaches the *hypothetical value* of n^2/α^2, i.e. ~ 137^2 (cf. Figure 3).

We can further elaborate on the ratio taking place at the RHS of Eq.(1):

$$\left(1-\frac{Ze^2}{m_{0\infty}r_0c_0^2}\right)\frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}}=\left(\frac{1}{1+Z^2\alpha^2}\right)\sqrt{1+Z^2\alpha^2}=\frac{1}{\sqrt{1+Z^2\alpha^2}}, \tag{14}$$

clearly showing that the *winning effect* is *(not the relativistic mass dilation, but)* the *proper mass decrease,* due to the binding.

On the other hand, the *total energy* E_{0n} of the nth level shall be calculated from

$$E_{0n}=-\frac{Ze^2}{r_0}+\left[m(r_0)-m_{0\infty}(r_0)\right]c_0^2=-\frac{Ze^2}{r_0}+m_{0\infty}c_0^2\frac{1-\frac{Ze^2}{m_{0\infty}r_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}-m_{0\infty}c_0^2\left(1-\frac{Ze^2}{m_{0\infty}r_0c_0^2}\right), \tag{15}$$

or

$$E_{0n}=-m_{0\infty}c_0^2\left(1-\frac{1}{\sqrt{1+\frac{Z^2\alpha^2}{n^2}}}\right). \tag{16}$$

(total energy written by the author, taking into account the overall mass decrease of the electron)

This quantity, decreases with increasing Z, to reach *asymtotically* the *floor* of $-m_{0\infty}c_0^2$. According to this approach, it is remarkable to note that, there is practically no *nucleus charge* no matter how big this can be, that can consume, *in its entirety*, the *proper mass* of the electron, through the *binding process*, we visualized (Figure 3). There is further an *inflexion point* of E_{0n} versus $Z\alpha$, at $Z\alpha = \sqrt{1/2}$.

Eq.(16), for small Z's, yields

$$E_{0n} \cong -\frac{m_{0\infty}c_0^2}{2}\frac{Z^2\alpha^2}{n^2}\left(1-\frac{1}{4}\frac{Z^2\alpha^2}{n^2}\right) \text{ (for small Z's).} \tag{17}$$

(approximate expression for the total energy written by the author, taking into account the overall mass decrease of the electron)

Via a similar equation to Eq.(15), set up within the frame of our approach [cf. Eq.(8)], where though, we *deliberately* overlook the *proper mass decrease*, just the way it had been so far conceived,† we can write the following *(the last part of which is approximated for small* Z's, *based on second order Taylor expansion):*

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† Eq.(8), via Eq.(9), in *this case* becomes

$$\frac{Ze^2}{r_0^2}\left(1-\frac{v_0^2}{c_0^2}\right) = m_{0\infty}\frac{v_0^2}{r_0}\ . \tag{i}$$

This can also be written as

$$\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}} = m\frac{v_0^2}{r_0}\ ; \tag{ii}$$

$$m = \frac{m_{0\infty}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\ . \tag{iii}$$

We have to consider Eq.(ii), together with Eq.(11), written for the *relativistic mass* m, i.e.

$$2\pi m v_0 r_0 = nh\ . \tag{iv}$$

Note that Eqs. (ii) and (iv), display *exactly* the same look as that of our *rigorous equations* Eqs. (10) and (11), were m taken for *(not the relativistic mass only, but, as originally)* the overall mass [cf. Eq.(1)]. Furthermore Eqs. (ii) and (iv), also display exactly the *same look* as that of the *set of equations written by Sommerfeld*, if now we *deliberately* wrote $m_{0\infty}$, instead of *the relativistic mass* m, thus yielding *(the Sommerfeld set)* $Ze^2/r_0^2 = \left(m_{0\infty}/\sqrt{1-v_0^2/c_0^2}\right)\left[v_0^2/c_0^2\right]$, and $2\pi m_{0\infty}v_0r_0 = nh$. This makes that both the *Sommerfeld set* and ours [i.e. Eqs. (ii) and (iv), furnish the *same velocity (which through the frame of concern, anyway turns out to be mass independent);* but Sommerfeld's radius, i.e. $r_{0n} = ne^2\sqrt{1+n^2/Z^2\alpha^2}/\left(\alpha m_{0\infty}c_0^2\right)$ is different from our radius [cf. Eq.(13)]. So are *(as elaborated in the text)*, the respective *total energies*. The *Sommerfeld set*, according to our approach is anyhow *incorrect*, not only because the grand master did not account for the *mass deficit of the bound electron*, but also he presumably considered the *Bohr postulate*, based solely on the *rest mass of the electron*, thence in contradiction with the *de Broglie's relationship*, suggesting instead, the use of the *relativistic mass of the electron*. It is further interesting to note that if Sommerfeld, next to his equation of motion, i.e. $Ze^2/r_0^2 = \left(m_{0\infty}/\sqrt{1-v_0^2/c_0^2}\right)\left[v_0^2/c_0^2\right]$, considered the Bohr's postulate by using *(not the rest mass, but)* the *relativistic mass of the electron*, to write $2\pi\left(m_{0\infty}/\sqrt{1-v_0^2/c_0^2}\right)v_0r_0 = nh$, then *he would have formally fallen back to the very original set of Bohr*, therefore locking himself in a situation where he can get no information about the *relativistic behavior of the electron.*

$$E_{0n}=-\frac{Ze^2}{r_0}+\left[m(r_0)-m_{0\infty}(r_0)\right]c_0^2=-\frac{m_{0\infty}v_0^2}{1-\frac{v_0^2}{c_0^2}}+m_{0\infty}c_0^2\frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}}-m_{0\infty}c_0^2$$

$$=-m_{0\infty}c_0^2\left(1+\frac{Z^2\alpha^2}{n^2}\right)\left(1-\frac{1}{\sqrt{1+\frac{Z^2\alpha^2}{n^2}}}\right)\cong-\frac{m_{0\infty}c_0^2}{2}\frac{Z^2\alpha^2}{n^2}\left(1+\frac{1}{4}\frac{Z^2\alpha^2}{n^2}\right)\quad . \tag{18}$$

(total energy, written by the author, taking into account the relativistic mass increase of the electron, but deliberately overlooking the proper mass decrease of the bound electron, yielding the Sommerfeld-Dirac result)

According to our approach this relationship is obviously *erroneous,* but we can consider it as a *tool* to check the *accuracy* of our derivation, at the sensivity level [10^{-5} x (the hydrogenen ionization energy)], in comparison with Dirac's result. Note anyway that the RHS of Eq.(18) well happens to be Sommerfeld's relativisitic result.[2] [Note further that *as known,* the Sommerfeld's result and the Dirac's result coincide regarding the *principal quantum numbers,* which makes that the RHS of Eq.(18), as we shall see below, also represents the Dirac's result, as well].

4. CHECKING THE ACCURACY OF OUR APPROACH

The total energy E_{Dirac}, taking into account both the *relativistic effect* and the *spin-orbit interaction,* given by Dirac, for nth *principal level,* is

$$E_{Dirac}=-\frac{m_{0\infty}c_0^2Z^2\alpha^2}{2n^2}\left\{1+\frac{Z^2\alpha^2}{n}\left(\frac{1}{j+\frac{1}{2}}-\frac{3}{4n}\right)\right\},\qquad j=l+\frac{1}{2}, \tag{19}$$

where l is the *quantum number associated with the angular momentum of the electron.*

In Dirac's theory, the *proper mass of the electron* in motion around the nucleus, being dilated due to the *Lorentz tranformation,* the *most probable distance of it, to the nucleus,* is decreased as much [for a quick check, cf. Eq.(2)].

For the ground state, for which n=1 (l=0), and the *spin-orbit interaction* term vanishes, the difference between the *classical Bohr's ground energy,* and *that of Dirac,* reduces to the *difference* ΔE_{Re1}, coming from *just* the *relativistic effect.*

This difference can be extracted from Eq.(19) to be

$$\Delta E_{Re1}=-\left|E_{0nB}\right|\frac{1}{4n^2}Z^2\alpha^2=-1.81\text{x}10^{-4}\text{ev for n=1 .} \tag{20}$$

(relativistic shift, as referred to Bohr' stotal energy, from Dirac's approach)

where we made use of the *magnitude* of the *classical Bohr's ground energy* $\left|E_{0nB}\right|$, at the nth level, i.e.

$$\left|E_{0nB}\right| = \frac{m_{0\infty}c_0^2}{2n^2} Z^2\alpha^2 = 13.6 \text{ ev for n=1, Z=1} . \tag{21}$$

We can propose to calculate the same quantity, through a *usual perturbation calculation*, based on the Schrodinger framework, presented in text books.[11]

This yields

$$\Delta E_{Rel} = -\left|E_{0nB}\right| \frac{5}{4n^2} Z^2\alpha^2 \ . \tag{22}$$

(relativistic shift, as referred to Bohr's total energy,
from the usual perturbation calculation,
based on the Schrodinger's framework)

Note that the *perturbation calculation* in question overestimates ΔE_{Rel} by a factor of *five*. Note further that we could have obtained the *same result* from a set up similar to Eq.(18), where we would evaluate both the potential energy and the Lorentz factor, based on the *straight* Bohr Atom Model.

On the other hand, our *Modified Bohr Theory, if just the relativisitic mass increase is considered (i.e. the mass decrease of the bound electron is not taken into account, then),* furnishes the same ΔE_{Rel} as that furnished by Dirac's theory [cf. Eqs. (19) and (20)].

Thus from Eq.(18) *(at the stage of which we have deliberately overlooked the mass decrease of the bound electron)* we have

$$\Delta E_{Rel} = -\left|E_{0nB}\right| \frac{1}{4n^2} Z^2\alpha^2 \ . \tag{24}$$

(relativistic shift as referred to Bohr's total energy,
from our Modified Bohr Atom Model, where
we have deliberately omitted the mass decrease)

Thus regarding ΔE_{Rel}, our *Modified Bohr Theory* has the capacity of furnishing *exactly* the *same result* as that furnished by Dirac solution, if geared alike.[‡]

---

‡ Note that the magnitude of our result ΔE_{Rel} [cf.Eq.(24)], can be *cross checked* immediately, via replacing *brutally* $m_{0\infty}$ of Eq.(21), by $m_{0\infty}\left[1 - Z^2\alpha^2/(2n^2)\right]$ [cf.Eq.(7)]:

$$\Delta E_{Rel} = -\left|E_{0nB}\right| \frac{1}{2n^2} Z^2\alpha^2 \ .$$

(from brutally replacing $m_{0\infty}$ *with our "overall mass",*
in the expression of the Bohr's total energy)

This quantity is *twice* as the *rigorous energy shift* we have calculated. However it still is closer to the *correct result* than that furnished by the perturbation theory [cf. Eq.(22)]. We will use this *outcome* below, to conclude that the *effect of mass decrease of the bound electron on the spin-orbit interaction* is about the same as the *effect of mass decrease of the bound electron on the classical relativistic correction.*

This can be accepted as a *check of the validity* and the *accuracy* of our *Modified Bohr Atom Model*, now toward a usage of it to assess *promptly* the effect of taking into account the *decrease of the proper mass of the bound electron,* together with the *relativistic effect,* this will display through its motion around the nucleus.

At this stage it would be interesting to sketch the electron's total energy in H atom, predicted by respectively Bohr [Eq.(21)], Sommerfeld-Dirac [(Eq.18)], and by ourselves [Eq.(16)], versus Z; this is done in Figure 4. Furhtermore we sketch in Figure 5, the correponding radii, in comparison with each other.

It is particularly interesting to note that, the magnitude of the total *energy we predict,* cannot go beyond $m_{0\infty}c_0^2$, no matter how big Z is. Whereas that predicted by Sommerfeld-Dirac, goes beyond $m_{0\infty}c_0^2$, even faster than that of Bohr, with respect to increasing Z's.

5. MASS DECREASE CORRECTION TO DIRAC'S EQUATION

According to our approach [cf. Eqs. (17) and (18)], we expect that, as referred to *Bohr's classical result,* the total energy should be shifted *upward,* and this, just as much as the *"downward shifting",* we would predict, via the *relativistic effect,* only [cf. Eq.(19)]; the *magnitude of the overall (upward) shift* ΔE_{shift} we predict, *as referred to the Dirac's theory,* thus $E_{0B}Z^2\alpha^2/(2n^2)$ [cf. Eqs. (6) and (7)].

Therefore, for n=1, $|E_{0B}|$ should be decreased altogether, as much as $|E_{0B}|Z^2\alpha^2/4$, i.e. 1.81x 10^{-4} ev.

We can thence bring a *first correction* to Dirac total energy, by *adding* to the RHS of Eq.(19), the quantity 2 x [the $|\text{RHS}|$ of Eq.(24)], i.e. $|E_0|Z^2\alpha^2/(2n^2)$:

$$E_{\text{Dirac (Corrected)}} \cong -\frac{m_{0\infty}c_0^2 Z^2\alpha^2}{2n^2}\left\{1-\frac{Z^2\alpha^2}{2n^2}+\frac{Z^2\alpha^2}{n}\left(\frac{1}{j+\frac{1}{2}}-\frac{3}{4n}\right)\right\},\ j=l+\frac{1}{2}\ . \qquad (25)$$

(the corrected Dirac energy, via taking into account the mass decrease of the electron, due to the binding)

We expect Eq.(25) to be valid for small Z's, for all n's, along with $l=0$.

Based on Eq.(25) we can now calulate the *energy shift* ΔE_{shift} *(upward),* for instance, for n=2: [§]

---

§ Recall that the shift in question, is a *correction* to be brought to Dirac's prediction; the corresponding shift referred to the classical Bohr's prediction, would be half of the first one.

$$\Delta E_{shift} = \left(\frac{m_{0\infty} c_0^2 Z^2 \alpha^2}{2n^2} \right) \frac{Z^2\alpha^2}{2n^2} = 13.6 \left(\frac{1}{137^2} \right) \frac{1}{32} \text{ev} \cong 2.3\text{x}10^{-5} \text{ev} \cong 0.182 \text{cm}^{-1}. \quad (26)$$

(energy shift correction for n=2*, to be brought to Dirac's solution, because of the mass decrease of the bound electron)*

This*, remarkably* points to the *right order of magnitude* of the *upward displacement* of the 2S level, conjectured already in 1937.[7]

Furthermore, based on Eqs.(25) and (26), one can immediately sense that $2^2 S_{1/2}$ and $2^2 P_{1/2}$ levels, *contrary to Dirac's theory, but in accordance with the experimental results,* differentiate from each other. This result is in harmony with the outlook of the Lamb shift[6] *(noticed more than half a century ago),* though *quantitatively,* as we will right away check, the measured shift between the $2^2 S_{1/2}$ and $2^2 P_{1/2}$ levels, i.e. 0.033 cm^{-1}, remains about *ten times* narrower than the outcome of Eq.(26).

The *order of magnitude of the shift* drawn by our approach, about the $2^2 S_{1/2}$ and the $2^2 P_{1/2}$ levels *(as referred to Dirac's theory)* can be assessed, based on a simple *perturbation calculation* to be achieved on the basis of *Dirac Equation,* where the *mass decrease of the bound electron* shall be incorporated. Or, in an even easier way *(leaving for the time being, aside the spin-orbit interaction),* we can consider *straight,* the Schrodinger equation, since the *energy shift* due to the *mass decrease effect* we focus on, should be expected to manifest practically in the same way, in both Dirac and Schrodinger descriptions, and it seems more practical to consider the second one.

However, as established above [cf. Eq.(22)], such a calculation, leads to a considerable overestimation *(roughly, by a factor of about five).*

Nonetheless, the perturbational method we propose, can still be considered as a fast *conventional mathematical tool* of demonstrating the fact that, the *mass decrease effect* indeed yields the *differentiation* of the $2^2 S_{1/2}$ and $2^2 P_{1/2}$ levels.

Thus here is the Schrodinger Equation, written *(with the familiar notation)* for the hydrogen atom, taking into account the *mass decrease of the bound electron:*

$$\nabla^2 \psi(\underline{r}_0) + \frac{8\pi^2 \mu_{0\infty}}{h^2} \left(1 - \frac{Ze^2}{\mu_{0\infty} r_0 c_0^2} \right) \left(E_0 + \Delta E_{shift} + \frac{Z_0 e^2}{r_0} \right) \psi(\underline{r}_0) = 0 \quad ; \qquad (27)$$

(Schrodinger Equation written by the author, for hydrogern atom, taking into account the mass decrease of the bound electron)

$\mu_{0\infty}$ is the *reduced mass* of the atom; r_0 the *distance of the electron* to the *center of mass of the electron and the proton;* $\psi(\underline{r}_0)$ is the *wave function* to be associated with the atom where the mass decrease is taken into account, and $\overline{\Delta E}_{shift}$ is the *total energy shift,* due to the *mass decrease of the bound electron,* in comparison with the classical *total energy* E_0.

$\overline{\Delta E}_{shift}$ can thus be calculated to be

$$\overline{\Delta E}_{shift} = \frac{\iiint_{space} \frac{(E_0 + |V|)}{\mu_{0\infty} c_0^2} |V| \psi^2_{0nljm_j} r^2 \sin\theta dr d\theta d\varphi}{\iiint_{space} \left(1 - \frac{|V|}{\mu_{0\infty} c_0^2}\right) \psi^2_{0nljm_j} r^2 \sin\theta dr d\theta d\varphi 0} \cong \iiint_{space} \frac{(E_0 + |V|)}{\mu_{0\infty} c_0^2} |V| \psi^2_{0nljm_j} r^2 \sin\theta dr d\theta d\varphi, \quad (28)$$

where $\psi_{0nljm_j}(\underline{r}_0)$, or in short ψ_{0nljm_j} is the *wave function* to be associated with the classical Schrodinger H atom, at the state described by the given quantum numbers, and $|V|$ is

$$|V| = \frac{Ze^2}{r_0}. \quad (29)$$

We would like to stress that $\overline{\Delta E}_{shift}$ of Eq.(28) is a *positive*, i.e. an *upward contribution.*

Eq.(28) yields

$$\overline{\Delta E}_{shift} = \frac{Z^2 |E_{0nB}| \alpha^2}{n} \left[2\left(\frac{2}{2l+1} - \frac{3}{4n} \right) - \frac{1}{\sqrt{n}} \sqrt{\frac{2}{2l+1} - \frac{3}{4n}} \right]. \quad (30)$$

(energy shift, upward, due to the mass decrease of the bound electron, calculated via perturbation)

Now, we can evaluate this quantity for the 2S (n=2, *l*=0) and 2P (n=2, *l*=1) levels:

$$\overline{\Delta E}_{shift}^{n=2,l=0} = \frac{Z^2 |E_{0nB}| \alpha^2}{2x4} \left(13 - \sqrt{13}\right), \quad (31)$$

$$\overline{\Delta E}_{shift}^{n=2,l=1} = \frac{Z^2 |E_{0nB}| \alpha^2}{2x4} \left(\frac{7}{3} - \sqrt{\frac{7}{3}} \right). \quad (32)$$

If there were *no other effects,* then the difference of these two quantities, would constitute the Lamb shift, *straight*. However, were the perturbation calculation we visualized *correct*, then the RHS result Eq.(31), should have been $Z^2 |E_{0nB}| \alpha^2 /(2n^2)$ [cf. (Eq.(26)]; thus, the RHS of Eq.(31) is overestimated by a *factor* of ($13 - \sqrt{13}$), i.e. 9.4.

Nonetheless this result is well in *conformity* with the *overestimation (by a factor of five)* of the perturbational result of the RHS of Eq.(22), and the *fact* that the *magnitude of the mass decrease effect is about twice that of the relativistic effect* [cf. Eqs.(6) and (7)], satisfactorily making that $9.4 \approx 5x2$.

Thus, despite this *inherent coherence,* the magnitude furnished by the perturbation calculation happens to be a *far off* in regards to the precision we would like to attain. However this calculation still clearly proves the *splitting* of the Dirac's *identical* 2S and 2P levels.

Anyhow recall that, as discussed through Section 4 above, we have, through our *Modified Bohr Approach,* well determined *seemingly* the *correct solution* that one would obtain by solving the Modified Dirac Equation, at least for *energy levels bearing just principal quantum numbers* [cf. Eqs. (24) and (25)].

Note that the *differenciation* of the two levels happens to be chiefly due to the the *shift* of $2^2S_{1/2}$ level, given that the *l* shift (for n=2), is about 10% of the corresponding s shift.

One other thing is that the *spin-orbit interaction term* too, should be affected by the *mass decrease of the bound electron.* However, this term vanishes for *l*=0; thus it has no effect on the 2S level shift. Moreover the magnitude of it, for higher *l's,* is much smaller, as compared to that of the *relativistic effect* to be associated with S levels, at a *given principal level.*

Note that our *corrected Dirac Equation,* i.e. Eq.(25), *even* in the case we should consider the *spin-orbit interaction term, within the frame of our original approach, consisting in the mass decrease of the bound electron*, still seems to be valid, and the reason is the following.

The relativistic correction ΔE_{Rel} to be brought to the *classical Schrodinger solution,* within the frame of a perturbation calculation, was recalled at the stage of Eq.(22), i.e.

$$\Delta E_{Rel} = -|E_{0nB}|\frac{5}{4n^2}Z^2\alpha^2 \quad (l=0) \ . \tag{22}$$

(from the usual perturbation calculation based on the Schrodinger's framework)

When a similar calculation is performed to evaluate the shift $\Delta E_{Spin-Orbit}$, that will be caused by the *spin-orbit interaction,* one comes out with[11]

$$\Delta E_{Spin-Orbit} = -|E_{0nB}|Z^2\alpha^2 \frac{[j(j+1) - l(l+1) - 3/4]}{2n\,l(l+1/2)(l+1)} \ . \tag{33}$$

The mass decrease of the bound electron, shall affect both quantities ΔE_{Rel} and $\Delta E_{Spin-Orbit}$, through the *parallel change* of E_{0nB}. These two quantites are then expected to be affected by the *mass decrease of the bound electron,* practically in the same amount *(regardless the fact that the rigorous result cannot be guessed, based on such a reasoning).*

This is why we affirmed that our *corrected Dirac Equation,* i.e. Eq.(25), in the case we consider the spin-orbit interaction term, together with the *mass decrease of the bound electron,* still seems to be valid. Thus, through the main stream of our approach, we can leave aside any further details related to the *spin-orbit interaction term,* that would result from the *mass decrease of the bound electron.*

At this point recall that the *discrepancies* between the *classical Dirac theory* and the *experimental results,* despite the huge effort displayed throughout more than half a century, are still not thoroughly resolved. Moreover, essentially speaking, what is kept in perpetual elaboration is the causes, already believed by then, which is fine for most of the components of the shift, but seemingly not sufficient.

Thus following our approach, our claim is that, *discrepancies* between the *classical Dirac theory* and the *experimental results,* such as *Lamb shift*, as well as the *fact that the doublets due to spin-orbit interaction are somewhat are narrower than predicted by Dirac,* are at least *partly* due to the *mass decrease of the bound electron.*

Recall that the *two leading believed causes of Lamb shift* are vacuum polarization[12] *(i.e. increase of the nuclear Coulomb interaction at small distances),* and self-energy[13, 14, 15, 16] *(also, a short-distance effect)*. Recall further that, apart the Dirac relativistic effect, and the Lamb effect, *other effects* such as the Breit contribution *(magnetic correlation effect),* the reduced mass effect[17] and the nuclear volume effect[18] *(due to the deviation from the point nucleus assumption),* had been given consideration, throughout.[19, 20, 21, 22, 23, 24]

Though, before one elucidates, *whether the mass decrease of the bound electron indeed indeed causes an upward shift of the Dirac's energy levels,* it seems useless to discuss the cumbersome details and the validity of the previous considerations and speculations.

Hence at this stage, we believe, what is to be fundamentally done, is *to decide whether or not a mass decrease comes into play, because of the binding of the electron to the nucleus;* only then, it seems rational to review in details, different effects together with their accurate contributions, and eventually attack, the general solution of the Modified Dirac's equation to be set following our approach.

As discussed in details through the introduction of this article, our claim is that, it is a *must* that the bound electron's *proper mass* decreases, and this is simply because of the special thoery of relativity *(more precisely because of the relativisitic equivalence between mass and energy).* But if so, we better modify our present conception about matter, and this is what we conclude with, below.

6. CONCLUSION: CHANGE OF THE METRIC NEARBY THE NUCLEUS

Our approach poses no doubt, problems about our actual conception of matter. What does it mean that the bound electron looses some of its *proper mass,* but preserves *its charged particle identity,* together with its original charge intensity?

However it may be, if so, based on the *equivalence between mass and energy,* one *immediate conclusion* we can retrieve from our approach, is that *(whatever it may be)* the *internal mechanism* of the *bound* electron slows down, in fact, just like the decay rate of a bound muon would retard.[25, 26, 27, 28, 29, 30]

Note that we were able to predict the *bound muon decay rate retardation (even before we had discovered in the literature, the related experimental data),* by the fact that, when bound, the internal dynamics of the muon *(basically composed of an electron, a neutrino and an antineutrino),* should get weaker, thus slower, and this just as much as the binding energy, the muon displays vis-à-vis the nucleus.[31]

Thus the muon, when bound, looses some of its proper mass, but preserves its *charged particle identity,* together with its *original charge intensity.*

We may conceive the *change* a bound electron undergoes, in a similar way.

Thus, whatever it may be, the bound electron's internal mechanism slows down, and this as much as the binding energy, the electron displays vis-à-vis the nucleus.

Accordingly, we can conceive the *"mass of the electron",* as just the *internal energy* of its *"charge",* however this *energy* is installed, and whatever the *"charge"* is.

One other interesting result our approach accordingly yields, is the *change of the metric near by the nucleus.*

We have indeed previously shown that, in any wave-like object, the *size of space* R_0 in which the object is installed, the *period of time* T_0, and the *characteristic mass* M_0, to be associated with the *internal dynamics* of the object are, owing to the *orchestration of electric charges,* interrelated *two by two,* in just a given manner, in fact in just the same way the simple Bohr Atom Model delineates.[32, 33, 34, 35, 36] This leads to the *invariance* of the closed form, $\mathsf{M}_0\mathsf{R}_0^2T_0^{-1}$, in regards to an *arbitrary change* in M_0, or the same, *the relativisitic total energy.*

It is fortunate that the quantity $\mathsf{M}_0\mathsf{R}_0^2T_0^{-1}$ also happens to be a *relativisitic invariant,* if the wave-like object, in hand were brought to a *uniform translational motion.* The same holds regarding the quantity $E_0\mathsf{M}_0\mathsf{R}_0^2$, where E_0 is the total energy of the wave-like object of concern.

Note further that the quantity $\mathsf{M}_0\mathsf{R}_0^2T_0^{-1}$, *quantum mechanically,* happens to be *strapped* to the *Planck Constant* h, just like the quantity $E_0\mathsf{M}_0\mathsf{R}_0^2$ happens to be strapped to h^2.

Thus, owing to the *orchestration of electric charges,* R_0 and M_0 are installed as *inversely proportional* to each other [for a hydrogenlike atom, for instance, cf. Eq.(2)]. *(Similarly* R_0 *and* T_0, *are installed proportionally to each other, and* R_0 *and* M_0, *are installed as inversely proportional to each other.)*

If R_0, M_0 and T_0, were quantities to be associated with the *internal dynamics of the electron,* in order to be in harmony with the wave-like property, we just mentioned, as well as the special theory of relativity, *"any change in the proper mass"* of the bound electron, would yield a *streching of the size of the electron,* and a *retardation of its internal mechanism,* no matter what this mechanism is, and how it is built.

Within the atom, one *fundamental unit length* can be considered as the *size of the electron;* likewise a *unit period of time* can be considered as the *period of the internal mechanism of the electron.*

Thus, the *change of the size,* as well as a *concurrent change of the period of time of the internal mechanism of the bound electron,* as referred to an outside observer, draws *a change in the metric,* within the atom, just like the *change in the metric induced by a gravitational field.*[5]

In other words a *local observer (or a local recording)*, and a *distant observer* shall assess, atomistic phemomena differently from each other. The difference is very small for small Z's, but becomes important as Z increases. Thus for instance, *our time* will be different than the *atomic time (the atomic time runs slower than our's),* and this is why we wanted to precise the subscript "$_0$" regarding the quantities we introduced, where we generally aimed to mean that, these quantities belong to the *local frame of reference,* and not our's.

It is amazing that the approach we presented herein can be applied to the motion of the celestial bodies, in a gravitational field as well, based on an equation similar to Eq.(1), along however exactly the same philosophy we developed, to predict very satisfactorily, *the end results* of the general theroy of relativity.[37, 38]

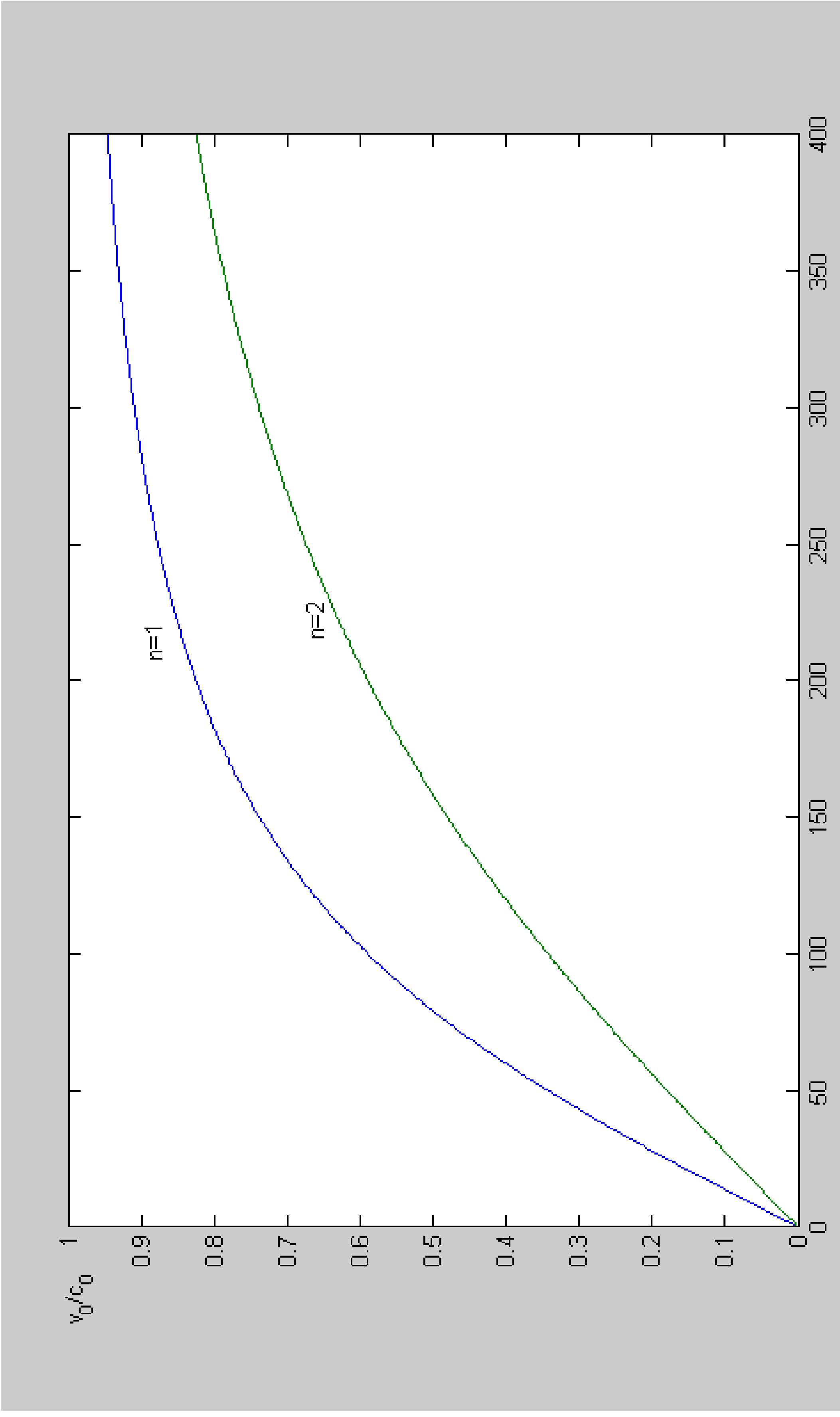

Figure 1 Orbit velocity in c_0 unit, for n=1 and n=2, with respect to Z

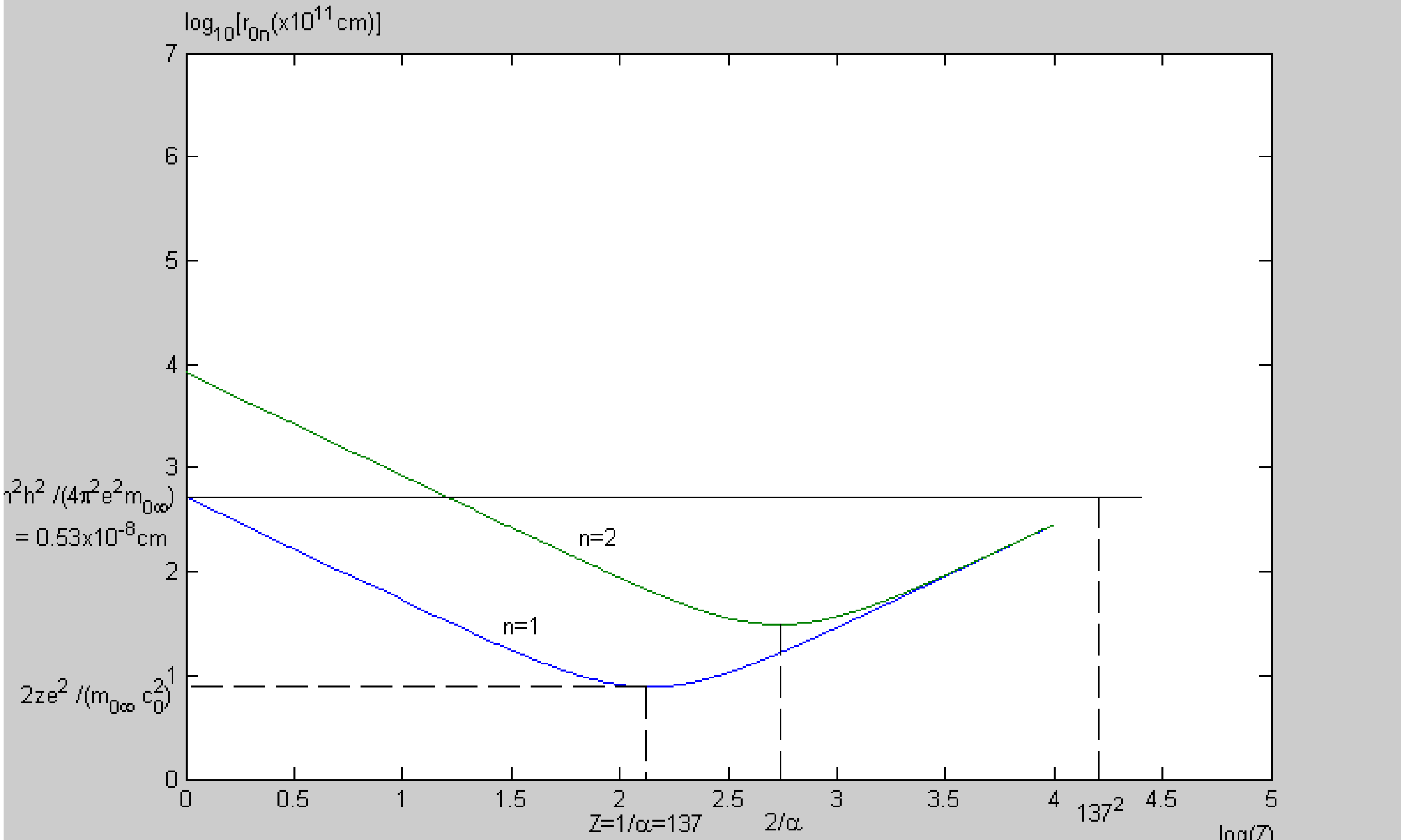

Figure 2 Orbit radius of the electron for n=1, and n=2, with respect to Z

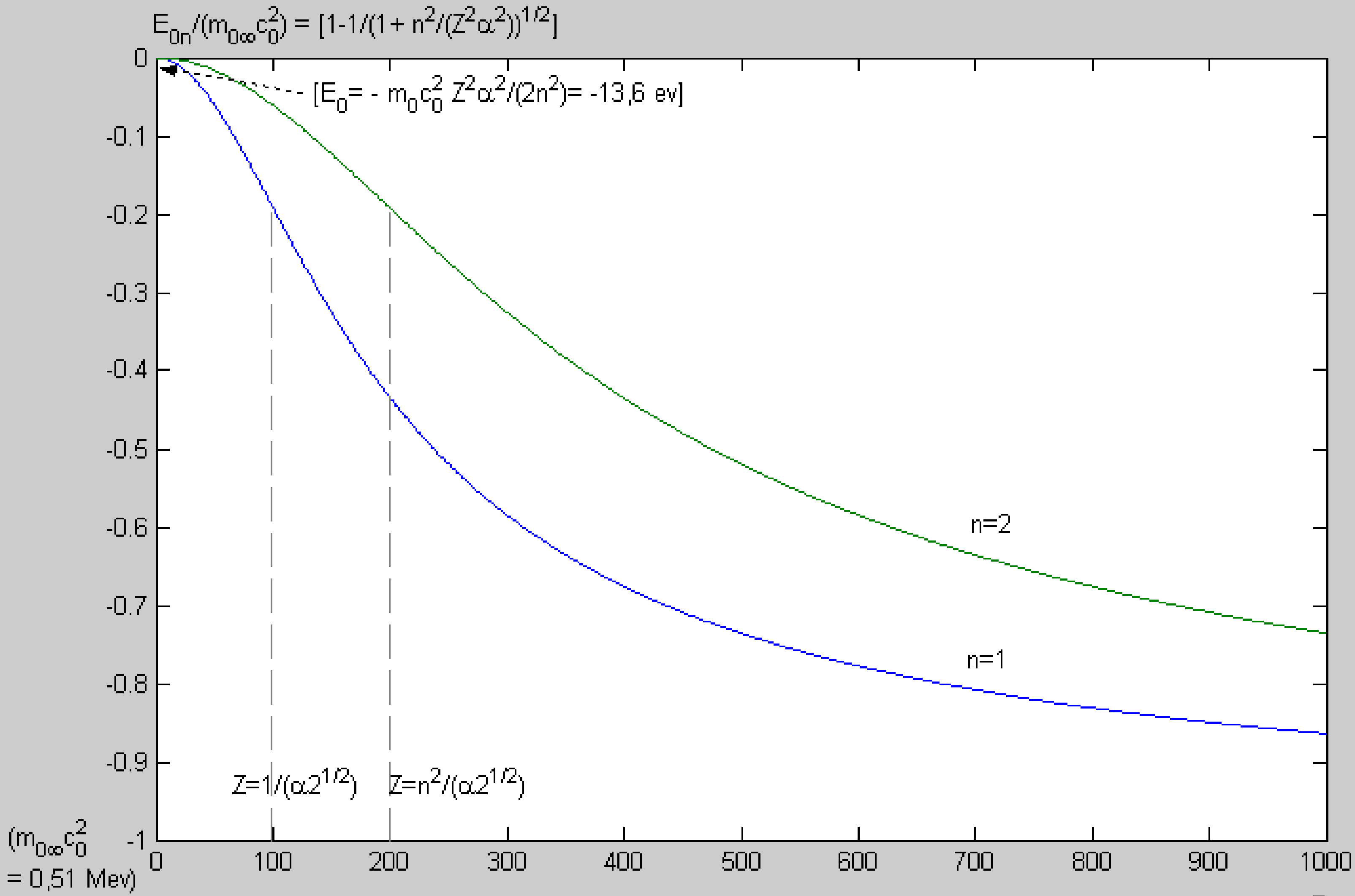

Figure 3 Total energy of the electron with respect to Z, for n=1 and n=2

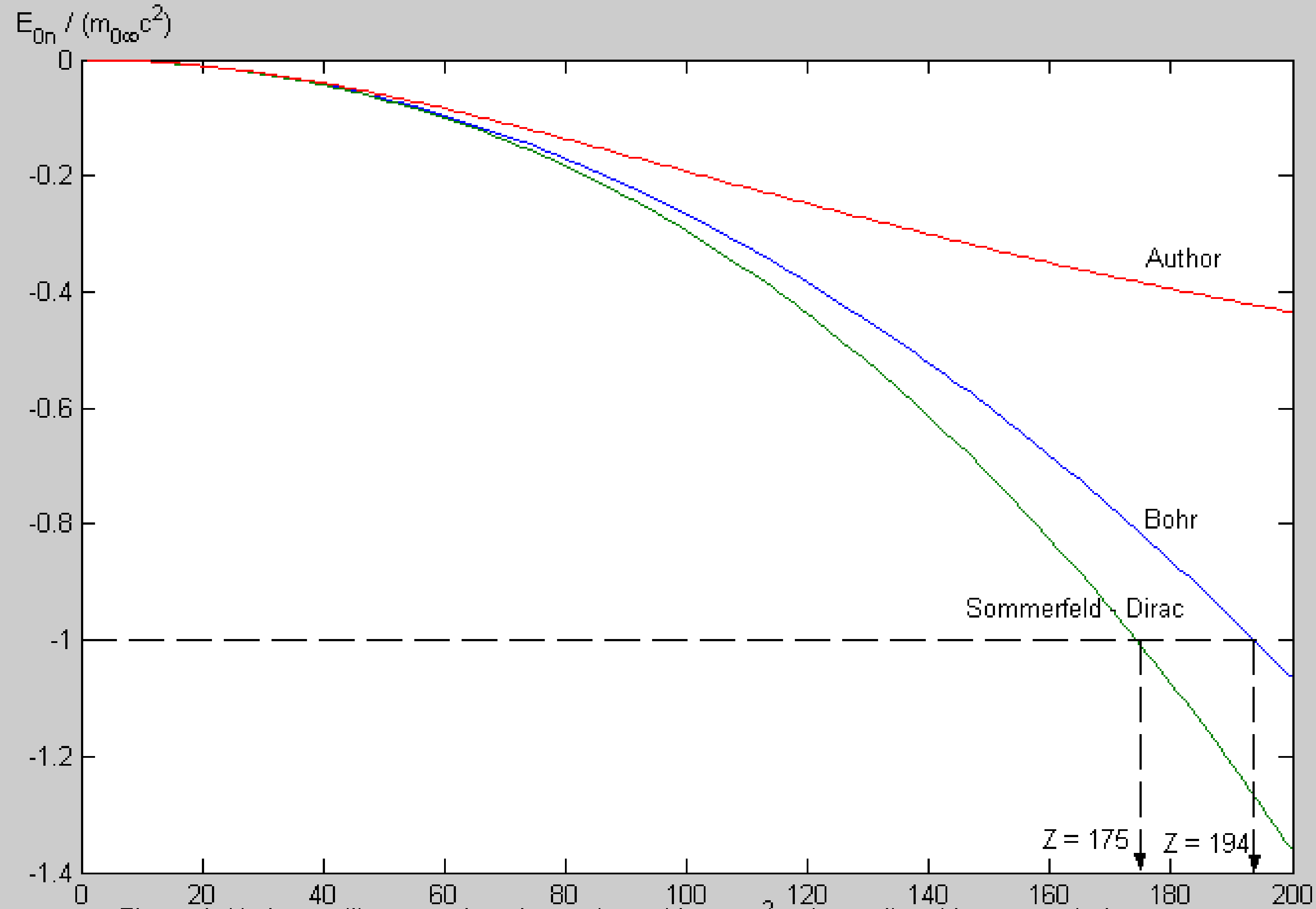

Figure 4 Hydrogen-like atoms' total energies, with $m_{0\infty}c_0^2$ unit, predicted by respectively Bohr, Sommerfeld-Dirac, and the Author's Atom Models, for n=1, with respect to Z

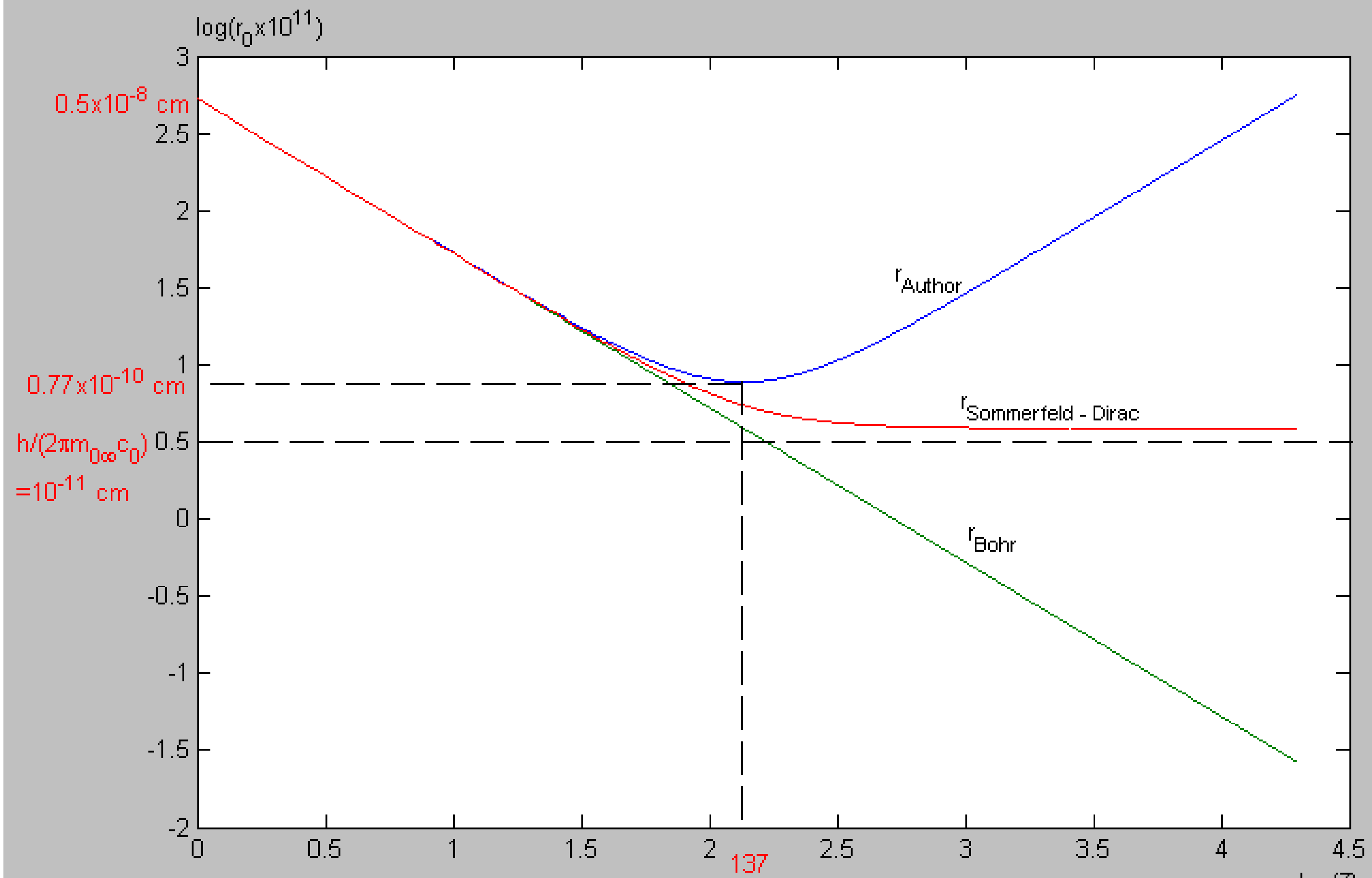

Figure 5 Hydrogen-like atom's radii, predicted by respectively Bohr, Sommerfeld-Dirac and the Author

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